

# Proportional Derivative – Type Iterative Learning Algorithm for a Motion Control System

Duong Thi Thanh Huyen <sup>a,1</sup>, Vu Van Hoc <sup>b,2,\*</sup>, Nguyen Thi Thanh Hoa <sup>c,3</sup>

<sup>a</sup> Thai Nguyen University of Technology, Thai Nguyen 250000, Vietnam

<sup>b</sup> Hanoi University of Science and Technology, Hanoi 100000, Vietnam,

<sup>c</sup> Hung Vuong University, Phu Tho 290000, Vietnam

<sup>1</sup> [duonghuyen-tdh@tnut.edu.vn](mailto:duonghuyen-tdh@tnut.edu.vn); <sup>2</sup> [vuvanhoc69@gmail.com](mailto:vuvanhoc69@gmail.com); <sup>3</sup> [nguyenthithanhhoa@hvu.edu.vn](mailto:nguyenthithanhhoa@hvu.edu.vn)

\* Corresponding Author

## ARTICLE INFO

## ABSTRACT

### Article history

Received March 30, 2023

Revised April 22, 2023

Accepted April 28, 2023

### Keywords

Machine learning control;

Controller;

Iterative learning algorithm;

Proportional Derivative

In this paper, Iterative Learning Control (ILC) combined with a Proportional Derivative (PD) regulator is proposed to deal with the problem of designing a control signal for motion control systems. The main idea in iterative learning control is to gradually improve the performance of the system by exploiting data from the previous iterations. The learning control algorithm can obtain a better tracking control performance for the next run and hence outperforms conventional control approaches such as Proportional Integral Derivative (PID) controller and feedforward control. The main area of application for ILC is control of industrial robots and CNC machine tool, printing, and other industrial applications. The learning algorithms can also be used in combination with other control techniques. For example, learning feedforward control is designed in the first iteration. Then iterative learning control is applied to improve performance in the subsequent iterations. In addition, the conventional feedback regulator is designed in combination with iterative control to deal with uncertainty. Simulation results demonstrate the potential benefits, sensitivity and robustness of the proposed method.

This is an open-access article under the [CC-BY-SA](https://creativecommons.org/licenses/by-sa/4.0/) license.



## 1. Introduction

Motion control is very popular in automated control systems. It provides the means to move the machine tooling or the part itself in a controlled and often precise, linear or rotary manner. It is an important part of robotics and CNC machine tools and is widely used in packaging, printing, textile, and other industrial applications. A motion controller contains the target positions and motion profiles for the application and creates the trajectories for the motor and/or actuator. Motion control is often a closed loop, so it monitors the actual path and corrects for position or velocity errors. They can be quite complicated because many different factors have to be considered in the design, such as reduction of the influence of plant disturbances, attenuation of the effect of measurement noise, and variations as well as uncertainties in plant behavior.

A conventional PID controller is first considered. For this type of controller, low PID gains are suggested to reduce the effect of measurement noise suggests, while high PID gains are recommended to attenuate the process disturbances. However, the requirements of reducing noises and disturbances

cannot be achieved simultaneously [1], [2]. This is the limitation of the PID controller. To overcome this problem, advanced controllers are proposed.

An adaptive control system is a system in which the structure and parameters of the controller can be changed according to the variation of the system so that the specified requirements are ensured [3], [4]. The model-reference adaptive system (MRAS)-based learning feedforward control (LFFC) aims to acquire the inverse dynamics of the plant. The idea of LFFC using MRAS-based adaptive components was proposed in [2], [5]. With feedforward control, the state-dependent disturbances can be compensated before they have time to affect the system. The control action for disturbance rejection is obtained from the feed-forward path output. The MRAS-based LFFC can be applied to arbitrary motion profiles.

Iterative learning control (ILC) has been an active research area for more than three decades. The paper of Arimoto and co-authors [8], [9], is often referred to as the main source of inspiration for research in this area. Several papers have also been written during this period [10]-[12]. The main idea of ILC is based on the notion that the performance of a system that executes the same task multiple times can be improved by learning from previous executions. In other words, ILC systems are applied to repetitive tasks over a specific finite time interval. The control input for a trial is updated using the information from the previous trial to iteratively compensate for complex interactions. This is an intelligent controller. It has the ability to learn, memorize and improve the quality in the next cycle. They have been very successfully applied in the robotics industry [13]-[16], calculating for CNC machines [17], moving wafers [18]-[20], injection molding machines [21], [22], metal casting machines [23], [24], cold rolling mills [25], [26], electromechanical valve actuator [27]-[29], locking brakes [30], [31], a Class of Distributed Parameter System [32]-[34] and many other fields [35]-[45].

The aim of this paper is to introduce an iterative learning control algorithm for motion control systems so that the tracking performance of the system using the MRAS-based LFFC will be improved. The reference trajectory of the mass is predetermined. Compared to a well-designed feedback and feedforward controller, ILC has several advantages [6], [11], [36]. A feedback regulator has to react to inputs and disturbances. Therefore, it always has a lag in transient tracking. This lag can be eliminated by a feedforward controller, but only for known or measurable signals. ILC is anticipatory and can compensate for exogenous signals, such as repeating disturbances, by learning from previous iterations. In ILC, the exogenous signals are not required to be known or measured but repeated from iteration to iteration. Since ILC cannot deal with unanticipated and nonrepeating disturbances, a feedback regulator is proposed to be used in combination with ILC.

The remainder of this paper is organized as follows: the dynamic characteristic of the setup is analyzed for testing the results of the controller in Section II. In order to eliminate positional inaccuracy due to reproducible disturbances and model uncertainty and systems with variable parameters and nonlinear behavior in the system dynamics, an MRAS-based LFFC controller is presented in Section III, and ILC is introduced in Section IV. Simulation results are shown in Section V. Finally, conclusions are given in Section VI.

## 2. Mathematical Model of the Setup

The setup designed for the purpose of testing the results of the controller is shown in Fig. 1. The mechanical part of the setup is designed to mimic printer technology. It consists of a slider that can move backward and forward over a rail. A DC motor, rail, and slider are fixed on a frame. A computer-based control system has been implemented with software generated by MATLAB [2]. Fig. 2 shows a second-order model of this setup.

The Damper component represents viscous and Coulomb friction. Coulomb friction always opposes the relative motion and is simply modeled as:

$$F_c = d_c \cdot \tanh(1000 \cdot \dot{x}) \quad (1)$$

where  $d_c$  is the Coulomb parameter of the Damper element,  $\dot{x}$  is the velocity of the load. Viscous friction is proportional to the velocity. It is normally described as

$$F_v = d. \dot{x} \tag{2}$$

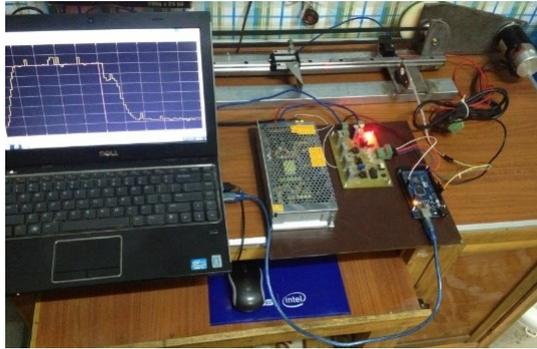


Fig. 1. The configuration of the setup

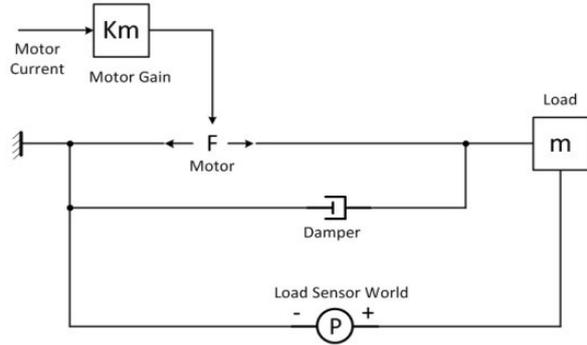


Fig. 2. Second-order model of the setup [2]

The mathematical expression for the combination of viscous and Coulomb friction is

$$F = F_v + F_c = d. \dot{x} + d_c. \tanh(1000. \dot{x}) \tag{3}$$

When the nonlinear friction term of the Damper element is disregarded, a second-order approximation model is obtained with a state-space description as given in Formula (4).

$$\begin{cases} \begin{bmatrix} \dot{v}_L \\ \dot{x}_L \end{bmatrix} = \begin{bmatrix} -\frac{d}{m} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_L \\ x_L \end{bmatrix} + \begin{bmatrix} -\frac{d_c}{m} \cdot \tanh(v_L) \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{k_m}{m} \\ 0 \end{bmatrix} F \\ y = [0 \quad 1] \begin{bmatrix} v_L \\ x_L \end{bmatrix} + [0]F \end{cases} \tag{4}$$

where  $v_L$  is the velocity of the load;  $x_L$  is the position of the load, and  $F$  is the applied force on the process.

### 3. PD regulator Combined with an MRAS-based LFFC

In order to eliminate positional inaccuracy due to reproducible disturbances and model uncertainty, a learning feed-forward controller structure that consists of feedback and a feedforward controller is considered [7]. The control structure, which consists of a PD regulator and an MRAS-based LFFC, was introduced in [9]. The approach seeks to benefit the systems with variable parameters and nonlinear behavior in the system dynamics. The detail of designing MRAS-based LFFC can be found in [2]. Fig. 3 shows the block diagram of this controller.

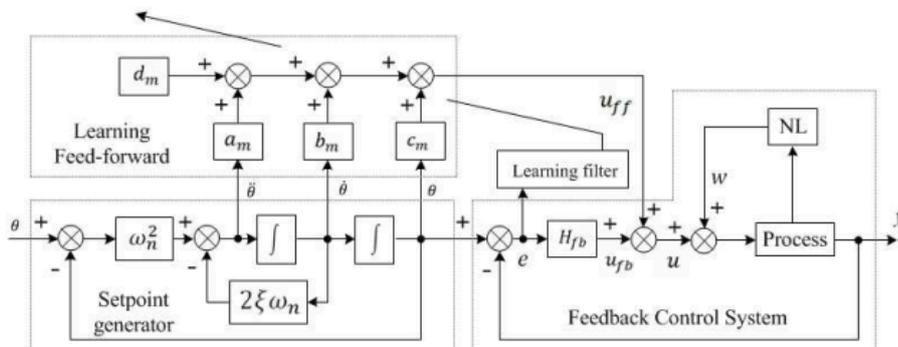


Fig. 3. LFFC combined with PD regulator [2].

As can be seen in Fig. 3, the learning signal is determined by the difference between the output of the reference model and the process. The feedback used to compensate for random disturbances thus also generates the learning signal for the learning mecha feed-forward parameters  $a_m, b_m, c_m$ .

and  $d_m$ , such that they converge to ideal values that cause the process response to match the response of the reference model [9].

#### 4. Iterative Learning Control

The main idea of ILC is to utilize the situation that the system to be controlled will carry out the same task over and over again. It will then be possible to gradually improve the performance of the control system by using the results from the previous iterations when choosing the input signal for the next iteration. The type of learning in ILC differs from other learning strategies, such as neural networks and adaptive controllers. That is, instead of learning the model or controller, which is a system, ILC learns the control input, which is a signal [6].

##### 4.1. Open Loop Iterative Learning Control

To illustrate the basic idea, an open-loop ILC is considered. For simplicity, we consider a servo problem and neglect the load disturbance [37], [40]-[45].

At iteration  $k$ , the output  $Y_k(s)$  is:

$$Y_k(s) = G(s) \cdot U_k(s) \quad (5)$$

The error signal:

$$E_k(s) = Y_D(s) - Y_k(s) \quad (6)$$

The basic structure of an ILC is shown in Fig. 4. The input signal  $U_k(s)$ ,  $G(s)$  denotes the transfer function and the error signal  $E_k(s)$  between reference trajectory  $Y_D(s)$  and system output  $Y_k(s)$  are stored in memory. The input signal for the next iteration is computed based on  $U_k(s)$  and  $E_k(s)$  to improve the system's performance. That is:  $U_{k+1}(s) = f(U_k(s), E_k(s))$ . In [6] and [7], the authors introduced some popular ILC design techniques that can be used in practical systems: PD-type design, plant inversion methods, quadratically optimal design (Q-ILC), and current iterative learning control. For simplicity, we consider the update equation:

$$U_{k+1}(s) = U_k(s) + H(s)E_k(s) \quad (7)$$

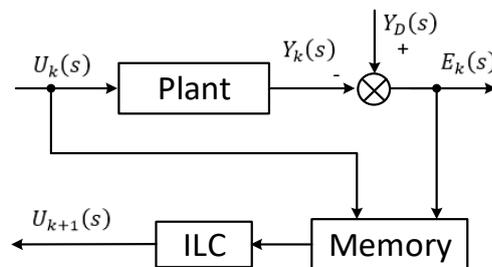


Fig. 4. Block diagram of basic iterative learning control.

where  $H(s)$  is a filter. Different choices of the filter  $H(s)$  have been discussed in the literature. The simplest form is, of course, to use a constant:

$$H(s) = K_p \quad (8)$$

In [9], the derivative of the error signal is used:

$$H(s) = K_d s \quad (9)$$

A combination of these two alternatives gives a PD-typed ILC algorithm:

$$H(s) = K_p + K_d s \quad (10)$$

Investigating what happens with the error signal when iterations continue, we consider the following:

$$E_{k+1}(s) = Y_D(s) - Y_{k+1}(s) = Y_D(s) - G(s) \cdot U_{k+1}(s) \quad (11)$$

Using [6], we have:

$$\begin{aligned} E_{k+1}(s) &= Y_D(s) - G(s)U_k(s) - G(s)H(s)E_k(s) \\ &= E_k(s) - G(s)H(s)E_k(s) \\ &= (1 - G(s)H(s))E_k(s) \end{aligned} \quad (12)$$

In the continuous-time open loop case, we see that with:

$$|1 - G(j\omega)H(j\omega)| < 1 \quad \forall \omega \quad (13)$$

The error will approach zero, and hence to output signal will follow the reference exactly. The condition in (13) means that the Nyquist diagram  $G(j\omega)H(j\omega)$  has to be inside a circle of radius one with the center at one. This circle is denoted learning circle.

#### 4.2. Closed Loop Iterative Learning Control

In this article, we consider an ILC in combination with conventional feedback, as discussed in [37]. Fig. 5 shows the proposed controller, which consists of ILC combined with a PD feedback regulator. The basic idea here is also that the system performs the same movement repeatedly, and a correction signal  $\Delta u_k$  is updated after each iteration.

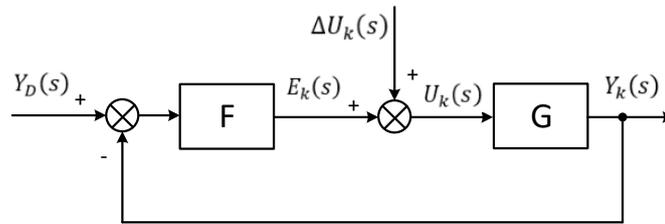


Fig. 5. ILC combined with a feedback regulator.

Different types of feedback may be included in this structure, and the most common case is a PD regulator. Alternative control methods, such as the state space approach, have also been used [38].

According to the block diagram, the input signal is now given by:

$$U_k(s) = F(s)(Y_D(s) - Y_k(s)) + \Delta U_k(s) \quad (14)$$

So, the output of the closed-loop system:

$$Y_k(s) = \frac{1}{1 + F(s)G(s)} (F(s)G(s)Y_D(s) + G(s)\Delta U_k(s)) \quad (15)$$

Using the output of the feedback regulator as an error signal, as shown in Fig. 5. That is,

$$E_k(s) = F(s) \cdot (Y_D(s) - Y_k(s)) \quad (16)$$

which using (15), gives:

$$E_k(s) = G_c(s)(G^{-1}(s)Y_D(s) - \Delta U_k(s)) \quad (17)$$

Where

$$G_c(s) = \frac{F(s)G(s)}{1 + F(s)G(s)} \quad (18)$$

is the transfer function of the closed-loop system.

Initially, we shall consider the same updating algorithm as in the open loop case. That is:

$$U_{k+1}(s) = U_k(s) + H(s)E_k(s) \quad (19)$$

Using (19), we obtained the following:

$$E_{k+1}(s) = G_C(s)G^{-1}(s)Y_D(s) - G_C(s)\Delta U_{k+1}(s) \quad (20)$$

which inserting (20) gives:

$$\begin{aligned} E_{k+1}(s) &= G_C(s)G^{-1}(s)Y_D(s) - G_C(s)\Delta U_k(s) - G_C(s)H(s)E_k(s) \\ &= E_k(s) - G_C(s)H(s)E_k(s) \end{aligned} \quad (21)$$

Hence:

$$E_{k+1}(s) = (1 - G_C(s)H(s))E_k(s) \quad (22)$$

Similar to the open loop case, we see that with:

$$|1 - G_C(s)H(s)| < 1 \quad \forall \omega \quad (23)$$

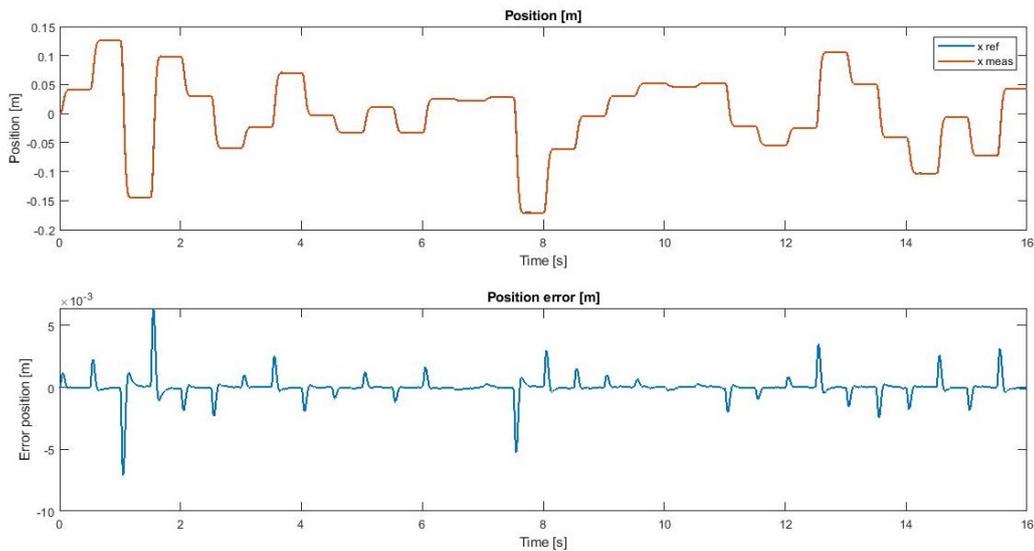
The error will approach zero. The condition is the same as for the open-loop case. The only difference is that the open loop transfer function  $G(s)$  has been replaced by the closed-loop transfer function.

## 5. Simulation results

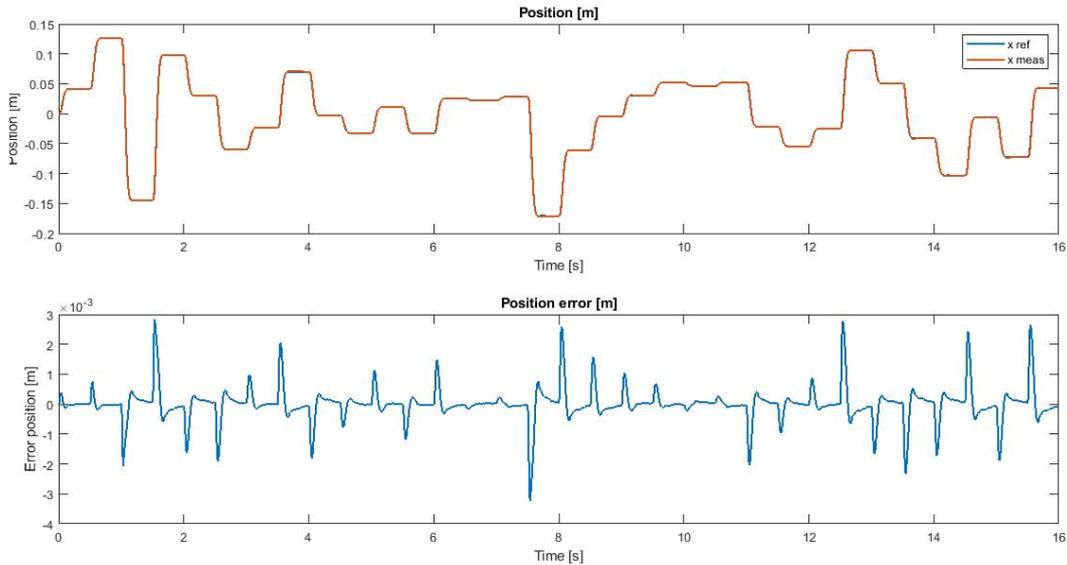
The Plant parameters are set up in Table 1. Fig. 6 and Fig. 7 show the simulation results of the feedback regulator and of the LFFC in combination with the feedback regulator. We can see that both controllers work well, but the LFFC combined with a PD regulator gives a better tracking performance, with a tracking error is less than  $3 \cdot 10^{-3}$  [m], while the tracking error of the PD-only controller is less than  $7 \cdot 10^{-3}$  [m].

**Table 1.** Plant parameters of the setup

Element	Parameter	Value
Motor-Gain	Motor constant	5.7 N/A
Frame	Mass of the frame	0.8 kg
Frame Flex	Spring constant	6 kN/m
Frame Flex	Damping in frame	6 Ns/m
Motor - Inertia	The inertia of the motor	$1e^{-5}$ kg
Load	Mass of slider	0.3kg
Belt Flex	Spring constant	80 kN/m
Belt Flex	Damping in belt	1 Ns/m
Damper	Viscous friction	3 Ns/m
Damper	Coulomb friction	0.5 N



**Fig. 6.** Simulation results of feedback regulator.

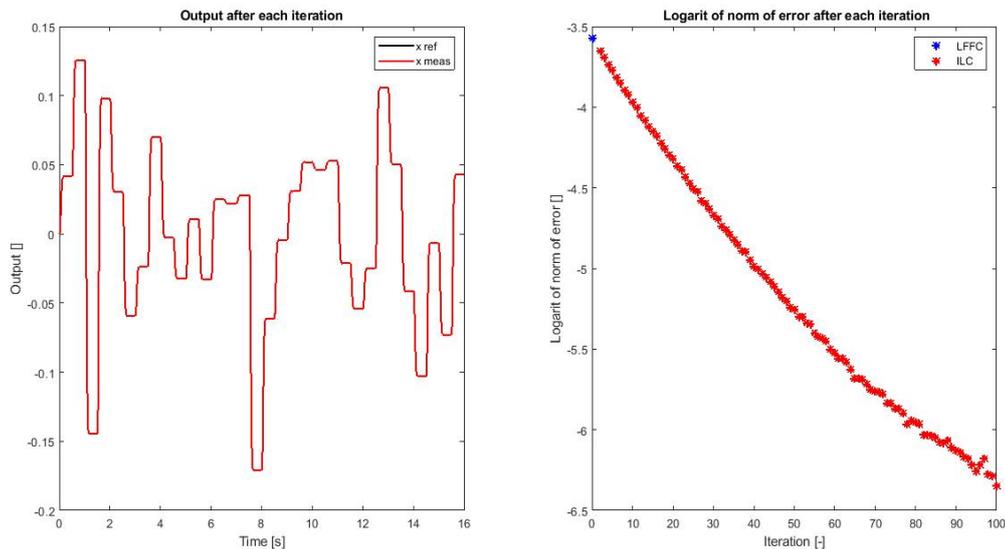


**Fig. 7.** Simulation results of LFFC in combination with feedback regulator.

The result from running LFFC is used as initial data for the ILC. The simulation result of the ILC combined with the PD regulator is shown in Fig. 8. To assess the control performance, the evolution of the norm of the tracking error over iterations is used, where:

$$\epsilon_{[i]} = \sum_{k=0}^{N-1} \|r_k - y_k\|_2. \quad (7)$$

This evolution can be influenced by  $K_p$  and  $K_d$  of the PD-typed ILC control algorithm. In the left figure, the output is almost overlapping the reference. In the right one, we can see that the norm of error decreases quickly after each iteration.



**Fig. 8.** Simulation results of ILC in combination with feedback regulator.

A major issue that needs to be considered when applying ILC is convergence. That is, the iterative update of the input signal converges to a signal giving good performance. The convergence aspects were discussed already in [10], where some convergence criteria were derived. In this project, the learning process can be stopped when the tracking performance is met. Fig. 9 shows the result of the last iteration. The figure indicates a very small tracking error, less than  $2 \cdot 10^{-4}$  [m]. In comparison

with the LFFC controller in the first iteration, the tracking error when using ILC is decreased almost more than times. This value can be improved for a larger number of iterations.

A disturbance is added to the system at  $t = 8$  [s]. In Fig. 6 and Fig. 7, we can see that disturbance increases tracking error. This disturbance in the ILC controller, however, is eliminated and does not affect the tracking error, as shown in Fig. 9. This is another advantage of the ILC controller.

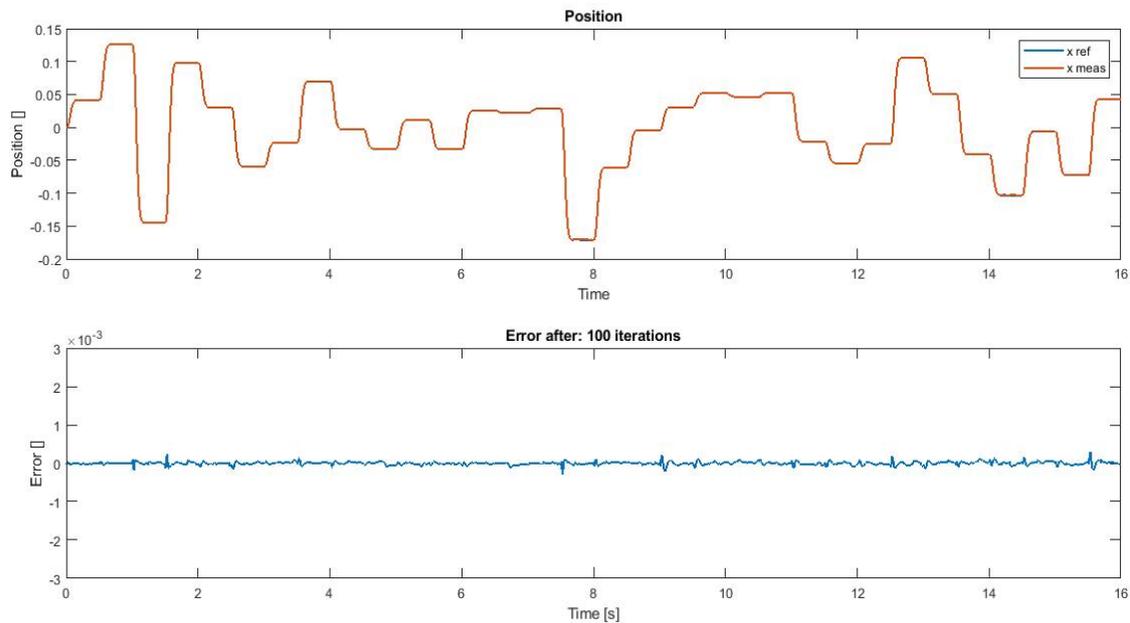


Fig. 9. Simulation of the last iteration of ILC.

## 6. Conclusion

In this paper, an introduction to the area of iterative learning control has been given. The basic principles behind the use of ILC in both open-loop and closed-loop control have been discussed. The main objective of this study is to apply iterative learning control design methodologies for motion control systems. Simulation results show that by learning from previous iterations, tracking performance is significantly improved. Since ILC cannot deal with unanticipated and nonrepeating disturbances, a feedback regulator is proposed to be used in combination with ILC. Improvement of robustness and improvement of the rate of convergence by using more complicated performance criteria. Results of iterative learning control simulation indicate that the algorithm possesses robustness to a useful degree. This issue is a topic of present research and will be addressed in future publications. The proposed ILC is not an optimal solution. Optimal ILC and implementation in the setup will be future works.

**Author Contributions:** Vu Van Hoc contributed to the control idea; Duong Thi Thanh Huyen participated in the control algorithm implementation, analysis of the simulation results, and paper setting; Nguyen Thi Thanh Hoa helped with the verification of the obtained system performance by discussing and paper correction.

**Acknowledgment:** The author would like to thank Vingroup JSC and Master, Ph.D. Scholarship Programme of Vingroup Innovation Foundation (VINIF), Institute of Big Data (VinBigdata).

**Funding:** This work was funded by Vingroup JSC and supported by the Master, Ph.D. Scholarship Programme of Vingroup Innovation Foundation (VINIF), Institute of Big Data (VinBigdata), code VINIF.2021.ThS.102.

**Conflicts of Interest:** The authors declare no conflict of interest.

---

**References**

- [1] D. C. Nguyen, "Advanced controllers for electromechanical motion systems" *Ph.D. thesis University of Twente*, 2008, [https://ris.utwente.nl/ws/portalfiles/portal/6039558/thesis\\_Cuong.pdf](https://ris.utwente.nl/ws/portalfiles/portal/6039558/thesis_Cuong.pdf).
- [2] N. D. Cuong, "Application of LQG combined with MRAS-based LFFC to electromechanical motion systems," *IFAC Proceedings Volumes*, vol. 46, no. 20, pp. 268-273, 2013, <https://doi.org/10.3182/20130902-3-CN-3020.00013>.
- [3] E. Lavretsky and K. A. Wise, *Robust Adaptive Control*. In: Robust and Adaptive Control. Advanced Textbooks in Control and Signal Processing. Springer, 2013, [https://doi.org/10.1007/978-1-4471-4396-3\\_11](https://doi.org/10.1007/978-1-4471-4396-3_11).
- [4] J. van Amerongen, "Intelligent Control (part 1)-MRAS," *Lecture Notes University of Twente*, 2004, [https://dynamicalsystems.nl/intelligentcontrol/Intelligent\\_Control\\_MRAS.pdf](https://dynamicalsystems.nl/intelligentcontrol/Intelligent_Control_MRAS.pdf).
- [5] J. van Amerongen, "An MRAS-Based Learning Feed-Forward Controller," *IFAC Proceedings Volumes*, vol. 39, no. 16, pp. 758-763, 2006, <https://doi.org/10.3182/20060912-3-DE-2911.00131>.
- [6] D. A. Bristow, M. Tharayil and A. G. Alleyne, "A survey of iterative learning control," *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96-114, 2006, <https://doi.org/10.1109/MCS.2006.1636313>.
- [7] D. Kim, H. Kim and K. Huh, "Local trajectory planning and control for autonomous vehicles using the adaptive potential field," *2017 IEEE Conference on Control Technology and Applications (CCTA)*, 2017, pp. 987-993, <https://doi.org/10.1109/CCTA.2017.8062588>.
- [8] T. J. A. De Vries, W. J. R. Velthuis, and J. van Amerongen, "Learning feed-forward control: A survey and historical note," *IFAC Proceedings Volumes*, vol. 33, no. 26, pp. 881-886, 2000, [https://doi.org/10.1016/S1474-6670\(17\)39256-X](https://doi.org/10.1016/S1474-6670(17)39256-X).
- [9] P. B. Dao, "Learning Feedforward Control Using Multiagent Control Approach for Motion Control Systems," *Energies*, vol. 14, no. 2, p. 420, 2021, <https://doi.org/10.3390/en14020420>.
- [10] X Bu, Z Hou, L Cui, and J Yang, "Stability analysis of quantized iterative learning control systems using lifting representation," *International Journal of Adaptive Control and Signal Processing*, vol. 31, no. 9, pp. 1327-1336, 2017, <https://doi.org/10.1002/acs.2767>.
- [11] K. L. Moore, M. Daleh, and S. P. Battacharrya, "Iterative Learning Control: A Survey and New Results," *Journal of Robotic Systems*, vol. 9, pp. 563-594, 1992, <https://doi.org/10.1002/rob.4620090502>.
- [12] R. Horowitz, "Learning Control of Robot Manipulators," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 115, pp. 403-411, 1993, <https://doi.org/10.1115/1.2899080>.
- [13] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123-140, 1984, <https://doi.org/10.1002/rob.4620010203>.
- [14] Y. M. Zhao, Y. Lin, F. Xi and S. Guo, "Calibration-Based Iterative Learning Control for Path Tracking of Industrial Robots," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 5, pp. 2921-2929, 2015, <https://doi.org/10.1109/TIE.2014.2364800>.
- [15] Y. -H. Lee, S. -C. Hsu, T. -Y. Chi, Y. -Y. Du, J. -S. Hu, and T. -C. Tsao "Industrial robot accurate trajectory generation by nested loop iterative learning control," *Mechatronics*, vol. 74, p. 102487, 2021, <https://doi.org/10.1016/j.mechatronics.2021.102487>.
- [16] S. Chen and J. T. Wen, "Industrial Robot Trajectory Tracking Control Using Multi-Layer Neural Networks Trained by Iterative Learning Control," *Robotics*, vol. 10, no. 1, p. 50, 2021, <https://doi.org/10.3390/robotics10010050>.
- [17] D. -I. Kim and S. Kim, "An iterative learning control method with application for CNC machine tools," *IEEE Transactions on Industry Applications*, vol. 32, no. 1, pp. 66-72, 1996, <https://doi.org/10.1109/28.485814>.
- [18] X. Fu, X. Yang, P. Zanchetta, Y. Liu, C. Ding, M. Tang, and Z. Chen, "Frequency-Domain Data-Driven Adaptive Iterative Learning Control Approach: With Application to Wafer Stage," *IEEE Transactions on Industrial Electronics*, vol. 68, no. 10, pp. 9309-9318, 2021, <https://doi.org/10.1109/TIE.2020.3022503>.
- [19] M. Li, T. Chen, R. Cheng, K. Yang, Y. Zhu and C. Mao, "Dual-Loop Iterative Learning Control With Application to an Ultraprecision Wafer Stage," *IEEE Transactions on Industrial Electronics*, vol. 69, no. 11, pp. 11590-11599, 2022, <https://doi.org/10.1109/TIE.2021.3120481>.

- [20] Y. Liu, L. Li, X. Yang, and J. Tan, "Enhanced kalman-filtering iterative learning control with application to a wafer scanner," *Information Sciences*, vol. 541, pp. 152-165, 2020, <https://doi.org/10.1016/j.ins.2020.05.125>.
- [21] D. N. C. Nam, N. M. Tri, H. G. Park, and K. K. Ahn, "Position control of electro hydraulic actuator (EHA) using an iterative learning control," *Journal of Drive and Control*, vol. 11, no. 4, pp. 1-7, 2014, <https://doi.org/10.7839/ksfc.2014.11.4.001>.
- [22] F. Gao, Y. Yang, and C. Shao, "Robust iterative learning control with applications to injection molding process," *Chemical Engineering Science*, vol. 56, no. 24, pp. 7025-7034, 2001, [https://doi.org/10.1016/S0009-2509\(01\)00339-6](https://doi.org/10.1016/S0009-2509(01)00339-6).
- [23] M. Pandit and K.-H. Buchheit, "Optimizing iterative learning control of cyclic production processes with application to extruders," *IEEE Trans. Contr. Syst. Technol.*, vol. 7, no. 3, pp. 382-390, 1999, <https://doi.org/10.1109/87.761058>.
- [24] A. A. Armstrong and A. G. Alleyne, "A multi-input single-output iterative learning control for improved material placement in extrusion-based additive manufacturing," *Control Engineering Practice*, vol. 111, p. 104783, 2021, <https://doi.org/10.1016/j.conengprac.2021.104783>.
- [25] S. S. Garimella and K. C. Srinivasan, "Application of iterative learning control to coil-to-coil control in rolling," *IEEE Transactions on Control Systems Technology*, vol. 6, no. 2, pp. 281-293, 1998, <https://doi.org/10.1109/87.664194>.
- [26] S. S. Saab, "A stochastic iterative learning control algorithm with application to an induction motor," *International Journal of Control*, vol. 77, no. 2, pp. 144-163, 2004, <https://doi.org/10.1080/00207170310001646282>.
- [27] W. Hoffmann, K. Peterson, and A. G. Stefanopoulou, "Iterative learning control for soft landing of electromechanical valve actuator in camless engines," *IEEE Transactions on Control Systems Technology*, vol. 11, no. 2, pp. 174-184, 2003, <https://doi.org/10.1109/TCST.2003.809242>.
- [28] M. F. Samadi and M. Saif, "Approach control of an electromechanical valve actuator using closed-loop iterative learning control," *2011 50th IEEE Conference on Decision and Control and European Control Conference*, 2011, pp. 5365-5370, <https://doi.org/10.1109/CDC.2011.6161223>.
- [29] N. R. Kapania and J. C. Gerdes, "Path tracking of highly dynamic autonomous vehicle trajectories via iterative learning control," *2015 American Control Conference (ACC)*, 2015, pp. 2753-2758, <https://doi.org/10.1109/ACC.2015.7171151>.
- [30] Y. Tang, X. Zhang, D. Zhang, G. Zhao, and X. Guan, "Fractional order sliding mode controller design for antilock braking systems," *Neurocomputing*, vol. 111, pp. 122-130, 2013, <https://doi.org/10.1016/j.neucom.2012.12.019>.
- [31] Z. Zhu and X. Zhang, "Spatial adaptive iterative learning control for high-speed train with unknown speed delays," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 2023, <https://doi.org/10.1177/09596518231155960>.
- [32] T. Xiao and H. -X. Li, "Eigenspectrum-Based Iterative Learning Control for a Class of Distributed Parameter System," *IEEE Transactions on Automatic Control*, vol. 62, no. 2, pp. 824-836, Feb. 2017, <https://doi.org/10.1109/TAC.2016.2571689>.
- [33] A. D. Barton, P. L. Lewin, and D. J. Brown, "Practical implementation of a real-time iterative learning position controller," *International Journal of Control*, vol. 73, no. 10, pp. 992-999, 2000, <https://doi.org/10.1080/002071700405941>.
- [34] N. T. Dzung, P. H. Phuc, N. Q. Dich, and N. D. Phuoc, "Iterative learning control for V-shaped electrothermal microactuator," *Electronics*, vol. 8, no. 12, p. 1410, 2019, <https://doi.org/10.3390/electronics8121410>.
- [35] C. Bo, L. Yang, Q. Huang, J. Li, and F. Gao, "2D multi-model general predictive iterative learning control for semi-batch reactor with multiple reactions," *J. Cent. South Univ.*, vol. 24, no. 11, pp. 2613-2623, 2017, <https://doi.org/10.1007/s11771-017-3675-6>.
- [36] E. Rogers, B. Chu, C. Freeman, and P. Lewin, *Iterative Learning Control Algorithms and Experimental Benchmarking*. John Wiley & Sons, 2023, <https://doi.org/10.1002/9781118535349>.

- [37] S. Tian, Q. Liu, X. Dai, and J. Zhang, "A PD-type iterative learning control algorithm for singular discrete systems," *Adv Differ Equ*, vol. 2016, no. 321, 2016, <https://doi.org/10.1186/s13662-016-1047-4>.
- [38] J. J. Craig, P. Hsu, S. S. Sastry, "Adaptive Control of Mechanical Manipulators," *The International Journal of Robotics Research*, vol. 6, no. 2, pp. 16-28, 1987, <https://doi.org/10.1177/027836498700600202>.
- [39] S. Gunnarsson and M. Norrlöf, "A short introduction to iterative learning control," *Linköping University*, 1997, <https://www.diva-portal.org/smash/get/diva2:316481/FULLTEXT02.pdf>.
- [40] M. Togai and O. Yamano, "Analysis and design of an optimal learning control scheme for industrial robots: A discrete system approach," *1985 24th IEEE Conference on Decision and Control*, 1985, pp. 1399-1404, <https://doi.org/10.1109/CDC.1985.268741>.
- [41] A. Zhang, J. Wei, L. Shi, D. Qin, and T. C. Lim, "Modeling and dynamic response of parallel shaft gear transmission in non-inertial system," *Nonlinear Dynamics*, vol. 98, pp. 997-1017, 2019, <https://doi.org/10.1007/s11071-019-05241-w>.
- [42] S. Tian, Q. Liu, X. Dai, and J. Zhang "A PD-type iterative learning control algorithm for singular discrete systems," *Adv Differ Equ*, vol. 2016, no. 321, 2016, <https://doi.org/10.1186/s13662-016-1047-4>.
- [43] L. Zhou, H. Tao, W. Paszke, V. Stojanovic, and H. Yang, "PD-Type Iterative Learning Control for Uncertain Spatially Interconnected Systems," *Mathematics*, vol. 8, no. 9, p. 1528, 2020, <https://doi.org/10.3390/math8091528>.
- [44] W. Paszke, E. Rogers, and M. Boski, "Repetitive process based design of PD-type iterative learning control laws," *Proceedings of the Mediterranean Conference on Control and Automation*, 2018, <https://doi.org/10.1109/MED.2018.8442499>.
- [45] D. Shen and X. Li, *Iterative Learning Control for Systems with Iteration-Varying Trial Lengths*. Springer, 2019, <https://doi.org/10.1007/978-981-13-6136-4>.