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Fuzzy Fault-Tolerant Control Applied on Two Inverted Pendulums with Nonaffine Nonlinear Actuator Failures

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ABSTRACT

This paper deals with the problem of fault-tolerant control for a class of perturbed nonlinear systems with nonlinear nonaffine actuator faults. Fuzzy systems are integrated into the design of the control law to get rid of the system nonlinearities and the considered actuator faults. Two adaptive controllers are proposed in order to reach the control objective and ensure stability. The first term is an adaptive controller involved to mollify the system uncertainties and the considered actuator faults. Therefore, the second term is known as a robust controller introduced for the purpose of dealing with approximation errors and exogenous disturbances. In general, the designed controller allows to deal automatically with the exogenous disturbances and actuator faults with the help of an online adaption protocol. A Butterworth low-pass filter is utilized to avoid the algebraic loop issue and allows a reliable approximation of the ideal control law. A stability study is performed based on Lyapunov's theory. Two inverted pendulum example is carried out to prove the accuracy of the controller.

1. Introduction

The last decade has witnessed plenty of control strategies in the area of control theory. The most famous of these control techniques is named fault-tolerant control. In fault-tolerant control, the main problem is how to design an adequate control law such that when a fault occurs from an arbitrary sensor and/or actuator, the controller will behave to keep stability and tracking performances in an acceptable functionality range. The main idea behind this condition is to make the controller immune against fault by fault isolation, detection or online reconfiguration.

As known in the literature, fault-tolerant control (FTC) can be classified into two huge categories, which are commonly named passive and active approaches (see [1, 2]). In the passive fault-tolerant control (PFTC), only a single controller with a fixed structure and/or parameters is developed to deal with all possible failure scenarios assumed to be known a priori. Hence, the passive technique does not require online fault detection, fault diagnosis, and control reconfiguration. In a real application, this technique is easy to implement, but it is more conservative [3-5]. Looking at the number of published works, many passive fault tolerant control have been developed in the literature for linear systems, a reliable H_∞ guaranteed cost control technique based on the linear matrix inequality (LMI) technique (see [6]). Moreover, a reliable H_∞ controller is designed (see [7, 8]) with the help of the

LMI approach and static output feedback control. Authors in [9] propose a reliable non-fragile H_∞ compensation filter, while in [10-12], fault-tolerant control techniques specifically for nonlinear systems are presented. Meanwhile, authors in [10] have proposed a combination of two robust controllers L_2/H_∞ with fuzzy static output feedback based on an iterative LMI approach. A reliable H_∞ filter is designed in [11] for a class of nonlinear networked systems using Takagi-Sugeno (T-S) fuzzy model and LMI technique. A static output feedback fuzzy controller is employed in [12] for T-S fuzzy systems with sensor faults based on LMI_S.

In [13], fault diagnosis combined with model predictive control was applied to a hybrid actuator. Whereas in [14], authors propose an active FT controller based on a Nussbaum-type function to deal with actuator faults. In [15], another active fault-tolerant control is proposed with the help of fuzzy systems and Nussbaum-type function under nonaffine nonlinear actuator faults. Besides, the Industrial robot arm has been investigated in [16], using optimal predictive control of a hybrid actuator with time-varying delay. Another interesting work has been investigated in [17], while an active FTC based on a backstepping technique is developed for a class of Multi-Input Multi-Output (MIMO) uncertain nonlinear systems. In this work, four kinds of velocity sensor faults, including bias, drift, loss of accuracy and loss of effectiveness, are considered. In [18], the authors-based feedback linearization and neural networks on developing a robust controller. On the other hand, without using fault-tolerant diagnosis, two links robot manipulator is controlled based on an indirect robust adaptive control scheme with fuzzy systems. In [19], the authors investigated an adaptive fault tolerant control for a class of nonlinear systems based on neural networks and implicit function theorem with unknown bias and loss of effectiveness actuator faults. Authors in [20] have synthesized an indirect adaptive fuzzy fault-tolerant scheme for a class of nonlinear systems with both actuator and sensor faults. A combination of fuzzy systems and backstepping techniques allowed the online estimation of all adaptive parameters and ensured the boundedness of all signals involved in the closed-loop system.

In this work, an active fault-tolerant control scheme is developed with the help of Fuzzy Logic Systems (FLSs). The uncertain dynamic is approximated along with the nonlinear nonaffine actuator faults, the proposed is robust against actuator faults and external disturbances, and it allows to deal with the approximation errors coming from the use of fuzzy systems. Butterworth low pass filter is incorporated to tackle the algebraic loop issue. Two inverted pendulum simulation example is presented to show the effectiveness of the control strategy. The main contribution in this work is five folds as

1) State-dependent-actuator fault is considered, instead of time-varying actuator fault [20], which gives a large immunity to the system against dependents faults.
2) The nonaffine control input issue is handled without any transformation or approximation.
3) The stability of the closed-loop system is guaranteed, and no further compact set is introduced for the tracking errors.
4) The considered control scheme allowed to deal automatically with actuator faults, while in [25-31], actuator faults are not at all considered in the design stage, which makes the system vulnerable to different kinds of faults.
5) The stability results show the global uniformly ultimate bounded GUUB of the closed loop system, while it is only partially uniformly ultimate bounded PUUB in [1-6, 25-31].

The remainder of the paper is organized as follows: Section 2 introduces the problem formulation. In Section 3, the controller methodology is developed and designed along with the stability analysis derivation. Extensive simulations on the two-inverted pendulum are given in Section 4. The conclusion is presented in Section 5.

2. Plant Model and Control Objective

2.1. Problem Formulation and Preliminaries

Consider the following class of MIMO nonaffine nonlinear systems described by

$$\sum_i \begin{cases} \dot{x}_{i1} = x_{i2} \\ \vdots \\ \dot{x}_{ij} = f_i(x, u_i) + d_i(t) \\ y_i = x_{i1}, \quad i = 1,2, \dots, q; \;\; j = 1, 2, \dots, n \end{cases} \tag{1}$$

Or equivalently

$$\sum_i \begin{cases} y_i^n = f_i(x, u_i) + d_i(t) \\ y_i = x_{i1}, \quad i = 1,2, \dots, q; \;\; j = 1, 2, \dots, n \end{cases} \tag{2}$$

where $x = \left[x_{1,1}, x_{1,2}, \dots, x_{1,n_1}, \dots, x_{q,1}, x_{q,2}, \dots, x_{q,n_q}\right]^T \epsilon \Re^n$ is the overall state vector, n_i is the order of the $i-th$ sub-system with $n = \sum_{i=1}^{q} n_i$; $u = \left[u_1, \dots, u_q\right]^T \in \Re^q$ is the system input vector; $d_i(t) \epsilon \Re^q$ denotes the exogenous disturbances vector; $y_i \in \Re^p$ is the output vector; $f_i, i = 1, 2, \dots, q$ are unknown smooth nonlinear functions. Let's add and subtract $u_i(t)$ in Equation (2), one can find

$$\sum_i \begin{cases} y_i^n = f_i(x, u_i) + d_i(t) + u_i(t) - u_i(t) \\ y_i = x_{i1} \end{cases} \tag{2a}$$

With some mathematical manipulations, we obtain the following:

$$\begin{cases} y_i^n = g_i(x, u_i) + d_i(t) + u_i(t) \\ y_i = x_{i1} \end{cases} \tag{2b}$$

where $g_i(x, u_i) = f_i(x, u_i) - u_i(t)$.

If we consider the expanded nonaffine nonlinear actuator faults model [5, 15, 20], it is given in Table 1.

Table 1. Expanded Actuator Faults

Actuator	Fault Kinds	Conditions	Fault Name
$u_i(t)$	$u_i(t) + \bar{u}_i(x,t)$	if $\bar{u}_i(x,t)$ is a constant	(Lock in place, also called Bias)
		$if\ \bar{u}_i(x,t) = \lambda t, \quad 0 < \lambda \ll 1$	(Drift)
		if $\bar{u}_i(x,t)$ is a nonlinear time-varying and state-dependent function	(state-dependent loss of accuracy)
	$\rho_i(x,t)u_i(t)$	$if\ \rho_i(x,t) = 1$	(Totally effective)
		$if\ \rho_i(x,t) = 0$	(Total loss of effectiveness)
		$if\ \rho_i(x,t)$ is a nonlinear time-varying and state-dependent function, where $\rho_i(x,t)\epsilon[0,1]$	(state-dependent loss of effectiveness)

Using the aforementioned definitions described in Table 1, one can write the faulty actuator as

$$u_i^f(t) = \rho_i(x,t)u_i(t) + \bar{u}_i(x,t)$$

Applying the faults model to Equation (2b), one can write

$$y_i^n = g_i(x, u_i) + d_i(t) + \rho_i(x,t)u_i(t) + \bar{u}_i(x,t) \tag{3}$$

Manipulating (3), we get the following result

$$y_i^n = g_i(x,u_i) + u_i(t) + f_{ai}(x,u_i) + d_i(t) \tag{4}$$

with $f_{ai}(x,u_i) = u_i(t) - \rho_i(x,t)u_i(t) - \overline{u_i}(x,t)$ is considered as a whole actuator faults model. With respect to the dynamic of the system (4), the following assumptions will be made

Assumption 1: the external disturbances $d(t)$ is bounded with a positive constant value $\bar{d}$ as

$$d_i(t) \le \bar{d}.$$

Assumption 2: the state vector is available for measurement.

The objective is to design an adaptive fuzzy controller for system (4) such that the system output $y_i(t)$ follows a desired trajectory $y_{di}(t)$ while all signals in the closed-loop system remain bounded.

Regarding the development of the control law, the following assumption should also be made:

Assumption 3: the desired trajectory $y_{di}(t)$and its time derivatives $y_d^{(i)}, i = 1, \dots, n$ are smooth and bounded.

Assumption 4: the approximation error is bounded as

$$|\varepsilon_i(x)| \le \bar{\varepsilon}_u$$

Now, let us define the tracking error vector as with

$$e_i(t) = y_{di}(t) - y_i(t) \tag{5}$$

The n-time derivative of the tracking error can be written as

$$e_i^n = y_{di}^n - y_i^n \tag{6}$$

$$e_i^n = y_{di}^{(n)} - g_i(x,u_i) - u_i(t) - f_{ai}(x,u_i) - d_i(t) \tag{7}$$

In the ideal case $d_i(t) = 0$ and $g_i(x,u_i), f_{ai}(x)$ are known, so the control objective can be achieved by choosing the ideal feedback linearization control law:

$$u_i = u_i{}^* = -v_i - g_i(x,u_i) - f_{ai}(x) + y_{di}^{(n)} \tag{8}$$

Applying the ideal control law, one can have

$$e_i^n = v_i$$

We can conclude that the chosen ideal control law stabilizes the system.

Since the nonlinear functions $g_i(x,u_i)$, $f_{ai}(x)$ are unknown, the ideal control law cannot be applied. In this situation, our goal is to approach this ideal control law using fuzzy systems.

Remark 1. It's important to point out that the chosen sensor and state-dependent actuator faults are commonly found in the literature (see Refs. [15, 17, 20, 23]). However, they are placed separately in the controller due to the difficulty to handle them all in the same controller. Although, in our proposed controller, time-varying sensor and state-dependent actuator faults are applied all applied in the studied system, which makes the design hard.

2.2. Universal Approximators

It is proven that fuzzy systems (FS) are capable of approximating any real continuous function on a compact set with arbitrary precision [21].

Let $x = [x_1, \dots, x_n]^T$ be the input of the FS and y its output. For each x_i is associated m_i fuzzy sets F_i^j in X_i its universe of discourse, as for $x_i \in X_i$ there is at least one degree of membership $\mu_{F_i^j}(x_i) \neq 0$ where $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m_i$.

The rules base of the FS has $N = \prod_{i=1}^{n} m_i$, fuzzy rules of the form:

R_k: $if\ x_1\ is\ \breve{F}_1^k\ \ and \dots and\ x_n\ is\ \breve{F}_n^k\ \ Then\ \ y = f_k(x), \qquad k = 1, \dots, N$

where $\breve{F}_i^k \in \{F_i^1, \dots, F_i^{m_i}\}$ are linguistic values and $f_k(x)$ is a numerical function of the output variable. In general, $f_k(x)$ is a polynomial function depending on input variables. If $f_k(x)$ is a polynomial of zero order, such as $f_k(x) = a^k$, we have the Takagi-Sugeno zero-order fuzzy system, which is used in this work. Each rule has a numerical conclusion, and the output of the FS is obtained by calculating a weighted average as given by the following relationship:

$$y(x) = w^T(x)\theta \tag{9}$$

with

- $\theta = [a^1 \ \ \dots \ \ a^N]$: values of the conclusions in the fuzzy rules.
- $w(x) = [w_1(x) \ \dots \ w_N(x)]^T$,

where

$$w_N(x) = \frac{\mu_k(x)}{\sum_{j=1}^{N} \mu_j(x)}\ , k = 1, \dots, N \tag{10}$$

with $\mu_k(x) = \prod_{i=1}^{n} \mu_{\breve{F}_i^k}$, $\breve{F}_i^k \in \{F_i^1, \dots, F_i^{m_i}\}$, which represents the weight of the rule R_k.

3. Controller Strategy and Stability Analysis

In this section, our interest is to approach the ideal control law to realize tracking of a given reference trajectory. To achieve these objectives fuzzy system is used to estimate the control law as a whole (direct approach). According to the property of the universal approximation [21] of fuzzy systems, the ideal control law can be approached by a fuzzy system of the form (9) as follows

$$u_i^* = w_i^T(x, u_{fi})\theta_i^* + \varepsilon_i(x) \tag{11}$$

With $\varepsilon(x)$the approximation error, $w(x, u)$ is a vector of fuzzy basis functions assumed properly set in advance by the user, and θ^* is somehow the vector of optimal parameters minimizing the function $|\varepsilon_i(x)|$.

$$\theta_i^* = argmin_\theta\{sup_x|u_i^* - w_i^T(x, u_{fi})\theta_i|\} \tag{12}$$

The filtered signal u_{fi} (Fig. 1) is defined as

$$u_{fi} = H_L(s)u_i \approx u_i$$

where $H_L(s)$ is the Butterworth low-pass filter (LPF).

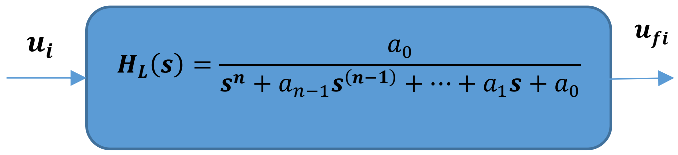

Fig. 1. Butterworth low-pass filter

Remark 2. It is worth noting that the unknown ideal control law $u_i^*(t)$ involve signals x_{ij}, x and u_i simultaneously. If the fuzzy logic systems $\hat{u}_i$ is directly used in controller strategy, an algebraic loop problem appears (see refs. [15, 17, 20, 23]). Hence, the filtered signal u_{if} is used instead of using directly u_i to avoid the algebraic loop problem in the fuzzy adaptive fault-tolerant control FTC. The replacement is logical since most actuators are low-pass properties.

The Butterworth filter parameters with a cutoff frequency $\omega_c = 1\, rad/s$ are depicted in Table 2.

Table 2. Parameters of the Butterworth filter (LPF).

Filter order (n)	a_4	a_3	a_2	a_1	a_0
2				1.414	1.000
3			2.000	2.000	1.000
4		2.613	3.141	2.613	1.000
5	3.236	5.236	5.236	3.236	1.000

We assume that the approximation error is bounded as follows:

$$|\varepsilon_i(x)| \leq \bar{\varepsilon}_u$$

where $\bar{\varepsilon}_u$ is an unknown positive constant. Note the terminal $\bar{\varepsilon}_u$ depends on the fuzzy basis functions selected. The more these functions are suitably chosen, the more terminal $\bar{\varepsilon}_u$ is small.

Since the optimal parameters $\theta_i{}^*$ are unknown, it is necessary to estimate the synthesis of the controller. Whether θ_i the estimate of $\theta_i{}^*$and will be calculated from an adaptation algorithm, besides the fuzzy adaptive approximation of ideal law is defined by

$$\hat{u}_i = w_i{}^T\theta_i + u_{ri} \tag{13}$$

Now we consider the following control law as

$$u_i = \hat{u}_i = w_i{}^T\theta_i + u_{ri} \tag{14}$$

Where u_{ri} is a robust control to get around the problem of approximation errors. It is defined by

$$u_{ri} = sign\left(e_i^T P_i B_i\right)\hat{\varepsilon}_{ti} - \sigma_i{}^2 \tag{15}$$

The estimation parameters use the following laws:

$$\dot{\theta}_i = \gamma e_i^T P_i B_i w_i(x) \tag{16}$$

$$\dot{\hat{\varepsilon}}_{ti} = n_t\left|e_i^T P_i B_i\right| \tag{17}$$

$$\dot{\sigma}_i = -\delta_0\sigma_i \tag{18}$$

when σ_i is a time-varying parameter with $n_f > 0, \gamma > 0\,, \delta_0 > 0$

Theorem 1. Consider the system (1), we assume that the assumptions 1-4 are satisfied. The control law defined by (14) with adaptions law (16-18) ensures the following properties:

- The tracking error and its derivatives converge to zero, $e_i^n(t) \to 0$ when $t \to\infty$ for $i = 0, 1, \ldots n-1$.
- The output of the system and its derivatives up to the order $(n-1)$ and the control signal are bounded: $y_i(t), \dot{y}_i(t), \ldots, y_i{}^{n-1}(t), u_i(t) \epsilon L_\infty$.

Proof 1.

$$e_i^n = y_{di}{}^{(n)} - y_i{}^{(n)} = y_{di}^{(n)} - g_i(x,u_i) - u_i(t) - f_{ai}(x,u_i) - d_i(t) + u_i{}^* - u_i{}^* \tag{19}$$

with $u_i{}^*$, it is the ideal control law

$$e_i^n = y_{di}{}^{(n)} - g_i(x,u_i) - f_{ai}(x,u_i) - d_i(t) + (u_i{}^* - u_i) - u_i{}^* \tag{19a}$$

Replacing Equations (8), (19a) becomes

$$e_i^n = -d_i(t) + (u_i{}^* - u_i) + v_i \tag{20}$$

$$u_i{}^* = w_i{}^T\theta_i{}^* + \varepsilon_{ui}(x) \tag{21}$$

$$u_i = \widehat{u_\iota} = w_i{}^T\theta_i + u_{ri} \tag{22}$$

Then

$$u_i{}^* - u_i = w_i{}^T\widetilde{\theta_\iota} + \varepsilon_{ui}(x) - u_{ri} \tag{23}$$

with

$$\widetilde{\theta_\iota} = \theta_i{}^* - \theta_i \tag{24}$$

$$e_i^n = -d_i(t) + \left(w_i{}^T\widetilde{\theta_\iota} + \varepsilon_{ui}(x) - u_{ri}\right) + v_i \tag{25}$$

Then the dynamics of the error can be written as follows:

$$\dot{e_\iota} = A_ie_i + B_i\left[\left(w_i{}^T\widetilde{\theta_\iota} + \varepsilon_{ui}(x) - u_{ri}\right) - d_i(t)\right] \tag{26}$$

with

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ -k_{ni} & -k_{ni-1} & \dots & -k_{ni} \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Until $(|sI - A_i|) = s^{(n)} + k_{1i}s^{(n-1)} + \cdots + k_{ni}$ is stable (A_i stable), we know that there exists a symmetric positive definite matrix P_i (n,n) that satisfies the Lyapunov equation:

$$A_i{}^TP_i + P_iA_i = -Q_i \tag{27}$$

where Q_i is a symmetric positive definite matrix of arbitrary dimensions $(n \times n)$.

Whether V_i is the Lyapunov function, then

$$V_i = e_i{}^TP_ie_i + \frac{1}{2\gamma}\widetilde{\theta_\iota}{}^T\widetilde{\theta_\iota} + \frac{1}{2n_t}\tilde{\varepsilon}_{ti}^2 + \frac{1}{2\delta_0}\sigma_i{}^2 \tag{28}$$

where $\tilde{\varepsilon}_{ti} = \bar{\varepsilon}_{ti} - \hat{\varepsilon}_{ti}$ and

$$\dot{V_\iota} = -e_i{}^TQ_ie_i + e_i{}^TP_iB_i\left[\left(w_i{}^T\widetilde{\theta_\iota} + \varepsilon_{u_i}(x) - u_{ri}\right) - d_i(t)\right] - \frac{1}{\gamma}\widetilde{\theta_\iota}{}^T\dot{\theta_\iota} - \frac{1}{n_t}\tilde{\varepsilon}_{ti}\dot{\hat{\varepsilon}}_{ti} + \frac{1}{\delta_0}\sigma_i\dot{\sigma_\iota} \tag{29}$$

$$\dot{V_\iota} = -e_i{}^TQ_ie_i + \dot{V_{1\iota}} + \dot{V_{2\iota}} \tag{30}$$

where

$$\dot{V_{1\iota}} = {e_i}^T P_i B_i {w_i}^T \widetilde{\theta_\iota} - \frac{1}{\gamma} \widetilde{\theta_\iota}^T \dot{\theta_\iota} \tag{31}$$

$$\dot{V_{2\iota}} = {e_i}^T P_i B_i \varepsilon_{ui}(x) - {e_i}^T P_i B_i u_{ri} - {e_i}^T P_i B_i d_i(t) - \frac{1}{n_t} \tilde{\varepsilon}_{ti} \dot{\hat{\varepsilon}}_{ti} + \frac{1}{\delta_0} \sigma_i \dot{\sigma_\iota} \tag{32}$$

Using Equation (16), then

$$\dot{V_{1\iota}} = 0 \tag{33}$$

Using assumptions 1 and 4, get

$$\dot{V}_{2i} \le |{e_i}^T P_i B_i| \bar{\varepsilon}_{ui} - {e_i}^T P_i B_i u_{ri} + |{e_i}^T P_i B_i| \bar{d} - \frac{1}{n_t} \tilde{\varepsilon}_{ti} \dot{\hat{\varepsilon}}_{ti} + \frac{1}{\delta_0} \sigma_i \dot{\sigma_\iota} \tag{34}$$

Let consider $\bar{\varepsilon}_{ti} = \bar{d} + \bar{\varepsilon}_{ui}$, using Equations (15-18), obtain

$$\dot{V_{2\iota}} \le 0 \tag{35}$$

Then the overall Lyapunov equation can be rearranged as follow

$$\dot{V_\iota} \le -{e_i}^T Q_i e_i \tag{36}$$

Finally, we can conclude that the Lyapunov-like equation is a decreasing function, so

$$\dot{V_\iota} \le 0 \tag{37}$$

Hence $V_i \epsilon L_\infty$, which implies that the signals $e_i(t)$, $\widetilde{\theta_\iota}(t)$, $\widetilde{\varepsilon_\iota}(t)$ and σ_i are bounded. This implies the boundedness of $x(t), u_i(t)$, and $\dot{e}(t)$, by using Babalat's lemma [1], we conclude that the tracking errors and their derivatives converge asymptotically to zero ${e_i}^{(n)}(t) \to 0$ when $t \to \infty$ for $i = 0,1,\dots,q$.

Remark 3. The proposed controller Eq. (14) with adaptive and robust Eq. (14) terms allow a fast compensation of the considered actuator faults without any observer and/or Fault detection and diagnosis method. Moreover, only the nonaffine structure in the system is handled without transformation and restricted assumption, which makes the proposed controller easy to implement in real applications. Only three adaptive parameters will be set online during the system operation (see Eq. (16, 17, 18)). In general, the studied nonlinear MIMO uncertain system Eq. (1) includes a real large application, i.e., Quadrotor, inverted pendulum, two inverted pendulums, spacecraft, ball and beam, and so on. The considered Lyapunov function and its derivative prove the stability of the overall closed system with respect to the considered actuator faults and the external disturbances. (For a better understanding of the controller scheme, you can refer to the overall diagram sown in Appendix A.

4. Simulation Results

To outline the effectiveness and benefits of the recommended direct Adaptive Fuzzy Fault-Tolerant Tracking Control (DAFFTTC), we consider the control issue of two-inverted pendulums associated with a spring as appeared in Fig. 2. Every pendulum might be situated by a torque input $u_i(t)$ connected by a servomotor at its base. Let $(x_{1,1}, x_{2,1}) = (\theta_1, \theta_2)$ be the angular positions of the pendulums from vertical and $(x_{1,2}, x_{2,2}) = (\dot{\theta}_1, \dot{\theta}_2)$ their angular velocities, respectively. The mathematical equations of the two-inverted pendulums are given by (see Ref. [15])

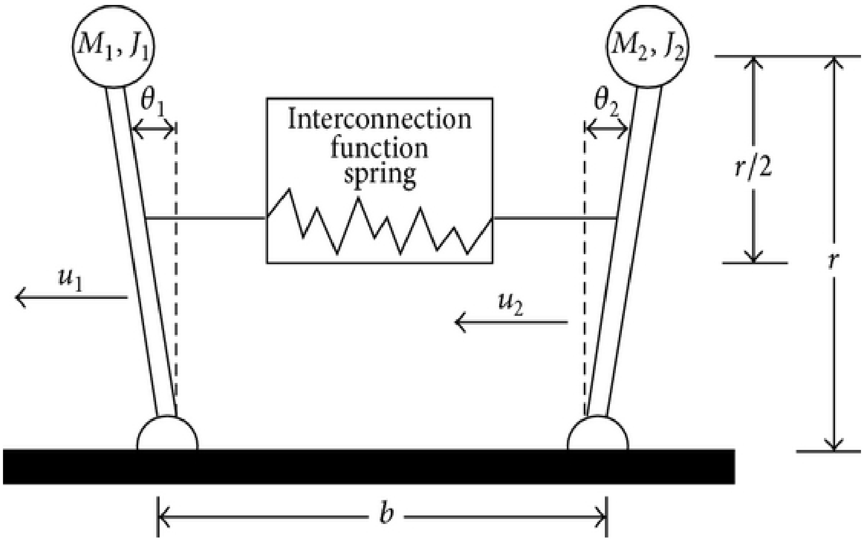

Fig. 2. Two-inverted pendulums configuration

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} \\ \dot{x}_{1,2} = \left(\frac{m_1 gr}{j_1} - \frac{kr^2}{j_1}\right) sin(x_{1,1}) + \frac{kr}{2j_1}(l-b) + \frac{kr^2}{4j_1} sin(x_{2,1}) + \frac{1}{j_1} u_1(t) + d_1(t) \\ \dot{x}_{2,1} = x_{2,2} \\ \dot{x}_{2,2} = \left(\frac{m_2 gr}{j_2} - \frac{kr^2}{j_2}\right) sin(x_{2,1}) + \frac{kr}{2j_2}(l-b) + \frac{kr^2}{4j_2} sin(x_{1,1}) + \frac{1}{j_2} u_2(t) + d_2(t) \end{cases} \tag{38}$$

Where m_1, m_2 the pendulum end-masses; j_1, j_2 the moments inertia; k the spring constant of the connecting spring; r the pendulum.

In this simulation, the control objective is to force the angular positions $y = [\theta_1, \theta_2]^T$ to track the desired trajectories $y_d = [\theta_{1d}, \theta_{2d}]^T$ under the simultaneous occurrence of four types of state-dependent actuator faults and disturbances.

The desired trajectories are selected as sinusoidal signals having the following equation:

$$y_d = [sin\,(t), sin\,(t)]^T \tag{39}$$

Remark 4. Contrasted to the works in the same area, we can easily see that the proposed desired trajectories have a maximum amplitude of 1 rad, while it is only limited to a small value (see Ref. [23]). Expanding the amplitude of the desired trajectories makes it more challenging to test the applicability of the proposed scheme controllers. Moreover, in our paper, the proposed control scheme is based on an online fuzzy logic system (FLSs) and Nussbaum-type function, which allows us to select these challenging trajectories.

Within this simulation, fifteen fuzzy systems in the form of Eq. (13) are used to approximate the unknown ideal control law u_i^*. The input variables of the used fuzzy systems are selected as $(x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2})$. For each input variable, we have defined five Gaussian membership functions with centers $C_i = [-3.5, -1.5, 0, 1.5, 3.5]$ and a variance equal to $\sigma = 1.6$.

$$\mu_{F_i^1}(x_i) = \exp\left\{-\frac{1}{2}\left(\frac{x_i - C_i}{\sigma}\right)^2\right\}, i = 1{:}4$$

The system initial condition is $x(0) = [\pi/6,0,\pi/6,0]$, sample time is $T_s = 0.01$, and the synthesis parameters of our controller and all adaption laws, and the physical parameters of the two-inverted pendulums are selected in Table 3 and Table 4, respectively. The parameters of the used two inverted pendulums are chosen according to [15, 24].

Table 3. Controller Parameters

Parameter (s)	Value(s)
γ, δ_0, η_t	2, 1.5
$\hat{\varepsilon}_{fi}(0)$ $i = 1:2$	0
$\sigma_i(0), i = 1:2$	1.5
$\theta_0(0) = 1:2$	0

Table 4. Two-inverted pendulum's physical Parameters

Parameter (s)	Value(s)
M_1	$2\ kg$
M_2	$2.5\ kg$
J_1	$0.5\ kg\ m^2$
J_2	$0.625\ kg\ m^2$
K	$100\ N\ m$
r	$0.5\ m$
b	$0.4\ m$
l	$0.5m$

The Butterworth low-pass filter function is given as

$$H_L(s) = 1/(s^2 + 1.414s + 1) \tag{40}$$

Within this simulation, a White Gaussian Noise (WGN) with centers $C_i = [0, 1.5]$ and a variance equal to $\sigma = 1.2$, applied on both angular positions and angular velocities. Three simulation cases are presented below to give a reliable test of the proposed control scheme with many scenarios.

The first simulation case is executed without any faults (free from actuator failures). Only disturbances are included. In Fig. 3(A) and Fig. 3(B), we can see the good tracking performances between the desired trajectories (y_{d1}, y_{d2}) and the angular positions (θ_1, θ_2), while Fig. 3(E) and Fig. 3(F) depict the corresponding tracking errors. Fig. 3(C) and Fig. 3(D) show the angular velocities. The control inputs of the two-inverted pendulums (u_1, u_2) are depicted in Fig. 3(G) and Fig. 3(H).

In the second simulation case, time-varying actuator faults (Bias, Drift, Loss of accuracy, Loss of effectiveness) and time-varying sensor faults are applied at the same time on the control inputs (u_1, u_2) at $T_f \geq 5s$. The considered faults are described in (see Table 5). In Fig. 4(A) and Fig. 4(B), we can see the good tracking performances between the desired trajectories (y_{d1}, y_{d2}) and the angular positions (θ_1, θ_2), while Fig. 4(E) and Fig. 4(F) depict the corresponding tracking errors. Fig. 4(C) and Fig. 4(D) show the angular velocities. The control inputs of the two-inverted pendulums (u_1, u_2) are depicted in Fig. 4(G) and Fig. 4(H).

Table 5. Time-varying actuator faults

Fault (s)	Type	Equation	Measurement unit
f_{ai}	Bias (Lock in place)	1	$[N.m]$
	Drift	$0.7 * t\ [N.m]$	$[N.m]$
	Loss of accuracy	$sin(t)\ + 0.7cos(t)[N.m]$	$[N.m]$
	Loss of effectiveness	87%	$[N.m]$

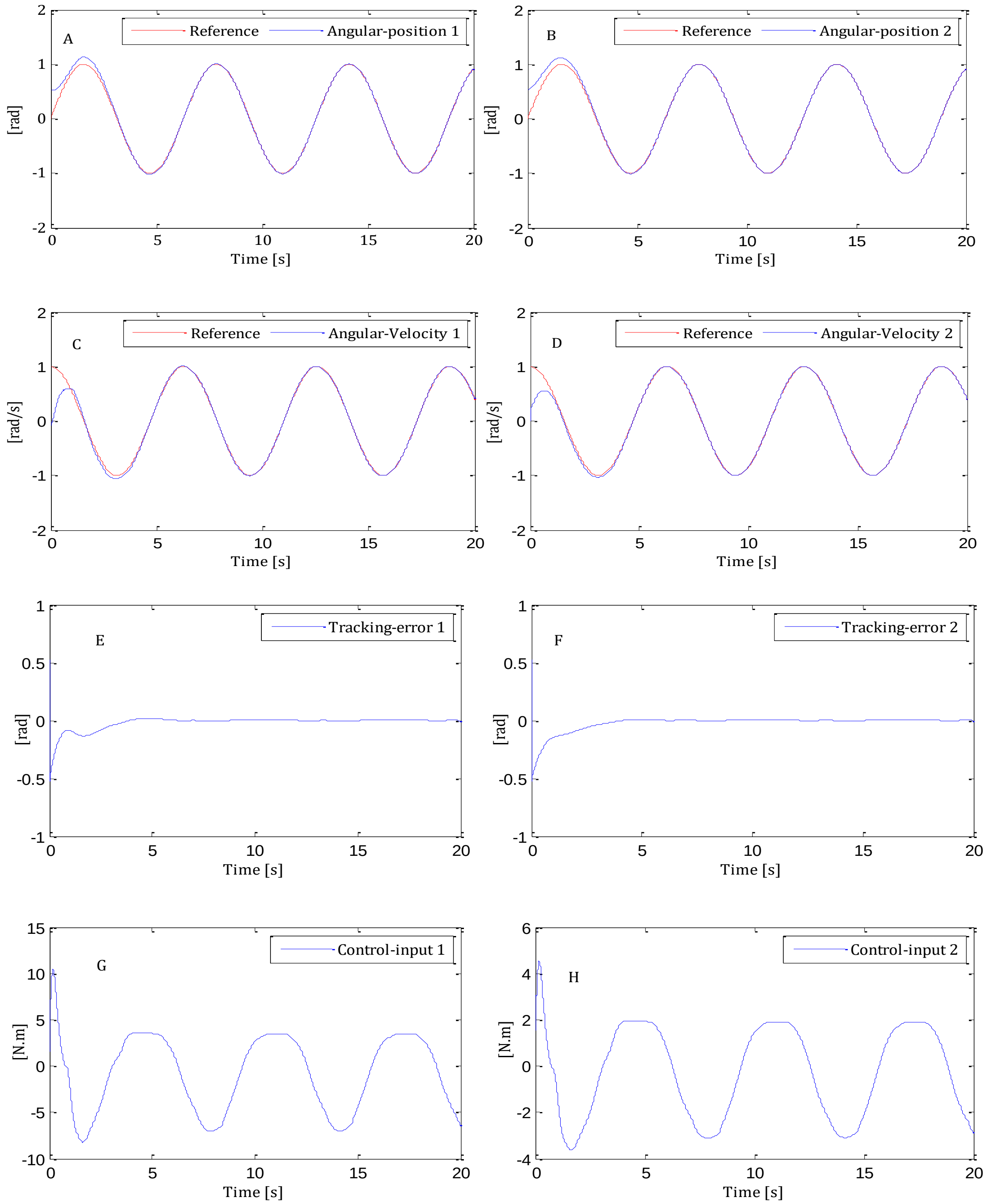

Fig. 3. Evolution of the two-inverted pendulums without any faults. (A, B) Trajectories-tracking of angular positions: actual (blue lines); desired (red lines); (E, F) Tracking error signal; (C, D) Trajectories-tracking of angular velocity: actual (red lines); desired (blue lines); (G, H) Control input signals.

Remark 5. In contrast to the work given in (see Refs. [15, 17, 20, 23]), when authors consider the faults for a short period of time during the simulation stage, only one or two kinds of faults are applied at the same time. Besides, in our simulation, four kinds of time-varying actuator faults (Bias, Drift, Loss of accuracy, Loss of effectiveness) are applied at the same time on the control inputs (u_1, u_2) at $T_f \geq 5s$, which allows us to ensure the desired performances (tracking and stability) during the whole simulation period.

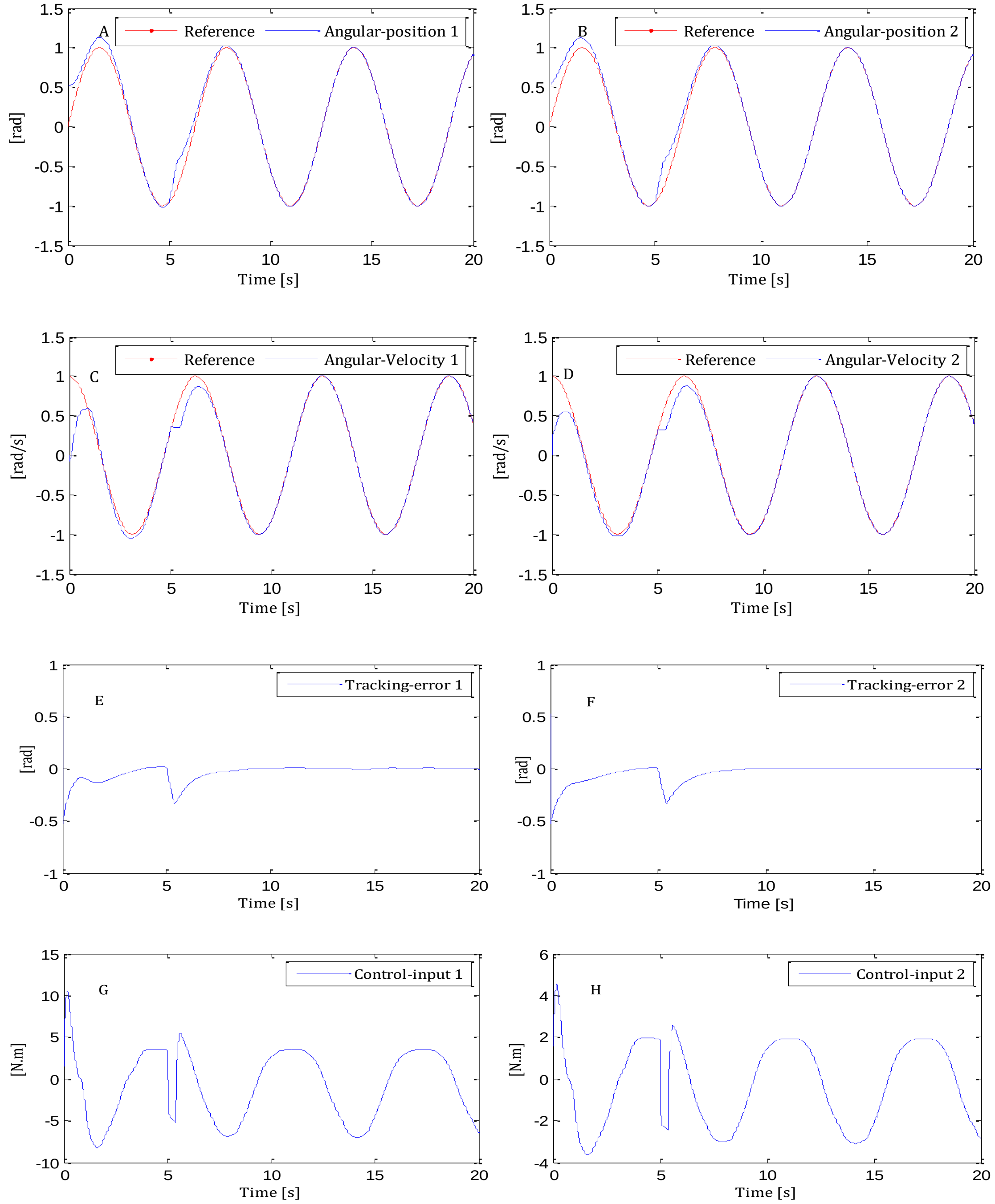

Fig. 4. Evolution of the two-inverted pendulums with Time-varying actuator faults. (A, B) Trajectories-tracking of angular positions: actual (blue lines); desired (red lines); (E, F) Tracking error signal; (C, D) Trajectories-tracking of angular velocity: actual (red lines); desired (blue lines); (G, H) Control input signals.

In the last case, the simulation study is carried out with time-varying and state-dependent actuator faults (see Table 1) and time-varying sensor faults at $T_f \geq 5s$. The form of the considered actuator faults is depicted in (see Table 6). In Fig. 5(A) and Fig. 5(B), we can see the good tracking performances between the desired trajectories (y_{d1}, y_{d2}) and the angular positions (θ_1, θ_2), while Fig. 5(E) and Fig. 5(F) depict the corresponding tracking errors. Fig. 5(C) and Fig. 5(D) show the angular velocities. The control inputs of the two-inverted pendulums (u_1, u_2) are depicted in Fig. 5(G) and Fig. 5(H).

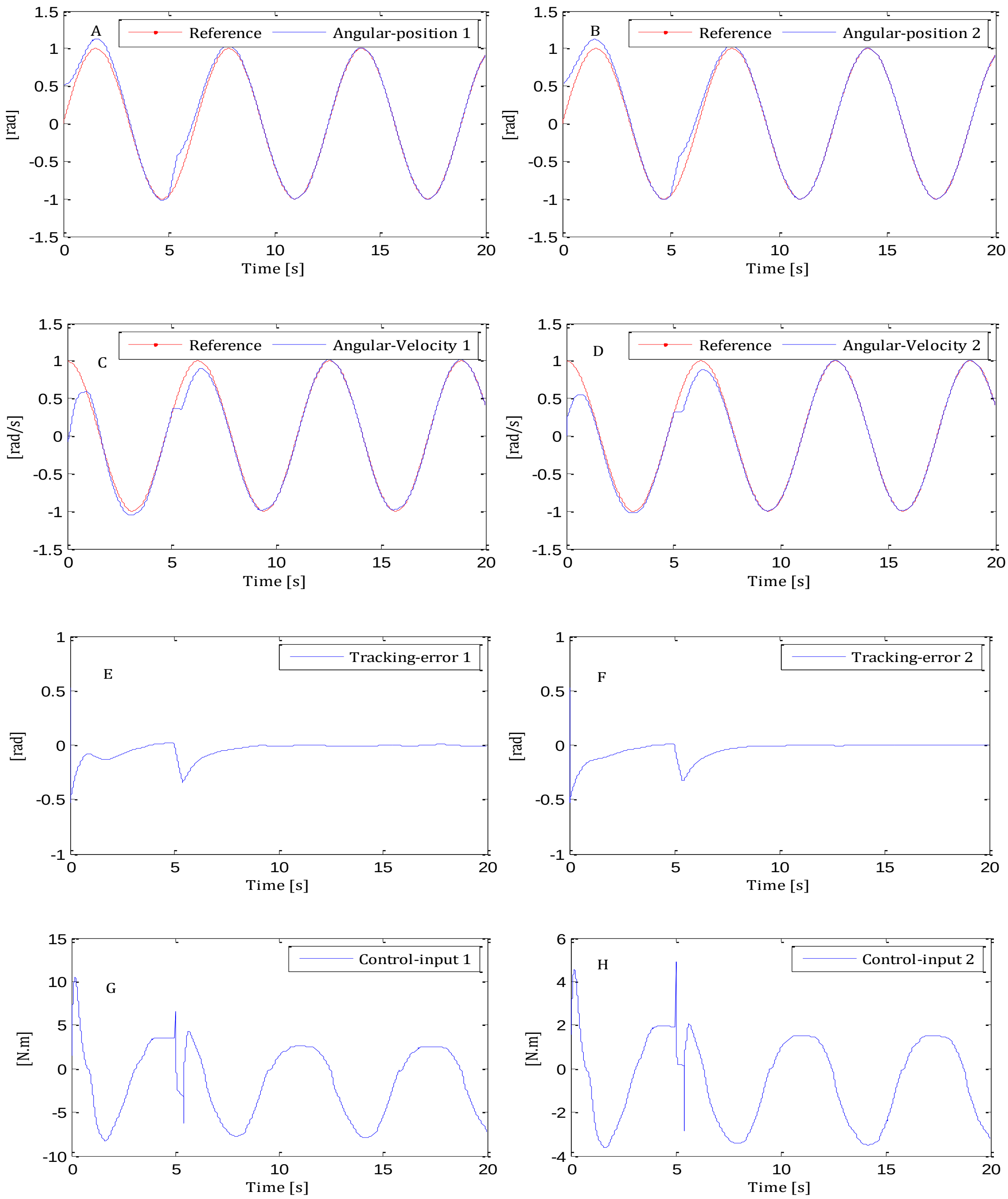

Fig. 5. Evolution of the two-inverted pendulums with Time-varying actuator faults. (A, B) Trajectories-tracking of angular positions: actual (blue lines); desired (red lines); (E, F) Tracking error signal; (C, D) Trajectories-tracking of angular velocity: actual (red lines); desired (blue lines); (G, H) Control input signals.

Table 6. Time-varying and state-dependent actuator faults

Fault (s)	Type	Equation	Measurement unit
f_{ai}	Bias (Lock in place)	3	$[N.m]$
	Drift	$0.7 * t$	$[N.m]$
	Loss of accuracy	$(4 + x_{1,1})cos(2t) - (2 + x_{1,2})sin(2t)$	$[N.m]$
	Loss of effectiveness	$(3 + \tanh(-t + 50x_{2,1})/10)/4$	$[N.m]$

4.1. Results Analysis and Comments

Depending on the performed simulation, the proposed control scheme leads to good transient performance against dynamics uncertainties, time-varying and nonaffine nonlinear state-dependent actuator faults, and tracking errors that converge asymptotically to the origin. The first case is faults-free, in which only disturbances are considered. Simulation results show good tracking performances with small tracking errors, and the applied control signal is smooth and bounded **(**see Fig. 3).

In the second case, time-varying sensor faults (see Table 5) are added at the same time for angles and angular velocities starting from $T_f \geq 5s$. We can easily remark that in the faulty instance, angle signals deviate from the desired trajectory signals by $0.1\ rad$. After almost $1\ sec$, the controller reacts quickly in order to compensate for the manifesting faults, which allows for fast fault compensation and keeps tracking performance. On the other hand, angular velocities deviate almost $0.1\ rad/s$. Furthermore, the applied effort during the faulty stage is limited to $3\ N.M$ for both first joint torque and second joint torque.

In the third case, nonaffine nonlinear state-dependent actuator faults (see Table 6) are added at the same time for the two motors (joint 1 and 2) starting from $T_f \geq 5s$. We can easily remark that in the faulty instance, angle signals deviate from the desired trajectory signals by $0.5\ rad$. After almost $4\ sec$, the controller reacts quickly to compensate for the manifesting faults, which allows for fast fault compensation and keeps tracking performance. On the other side, angular velocities deviate two times at $t = 5s$ and $t = 6s$. In the first time, when faults occur, angular signals change by $4\ rad/s$ and $5\ rad/s$. The reason behind the second reaction at $t = 6s$ is due to the considered nonaffine nonlinear faults, and then the controller takes almost $4s$ to resume the trajectories. The applied effort during the faulty stage $[5s, 12s]$ is limited to $5\ N.M$ for both motors.

5. Conclusion

Direct adaptive fuzzy fault-tolerant tracking control DAFFTTC is presented for a class of unknown multi-input multi-output MIMO nonaffine nonlinear systems. The advantages behind this approach compared with published papers in the same area are the capability for estimating the ideal control law with uncertainties and nonaffine nonlinear faults without any transformation and fault detection and diagnosis module. Only three adaptive parameters are set online, which gives a fast controller calculation burden and remain it easy to implement in real applications. The given robust control term allows a fast compensation of the errors due to the use of fuzzy systems, and the exogenous disturbances issue is handled theoretically instead of approximation. The theoretical development has not required the exact mathematical model of the plant. Even though the boundedness of all signals in the closed-loop system is guaranteed and the tracking error is converged to the origin. The main contribution of this paper is to find a solution on how to make the nonaffine system into an affine system and integrate a Butterworth low pass filter in the controller design to eliminate the algebraic loop problem issue. Moreover, the simulation example is challenging compared to other work in the field of the fault-tolerant control area. Further development can be achieved in future works by the integration of Metaheuristics approximation, like particle swarm optimization PSO in order to make the controller more precise during the transitory stage.

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Appendix A

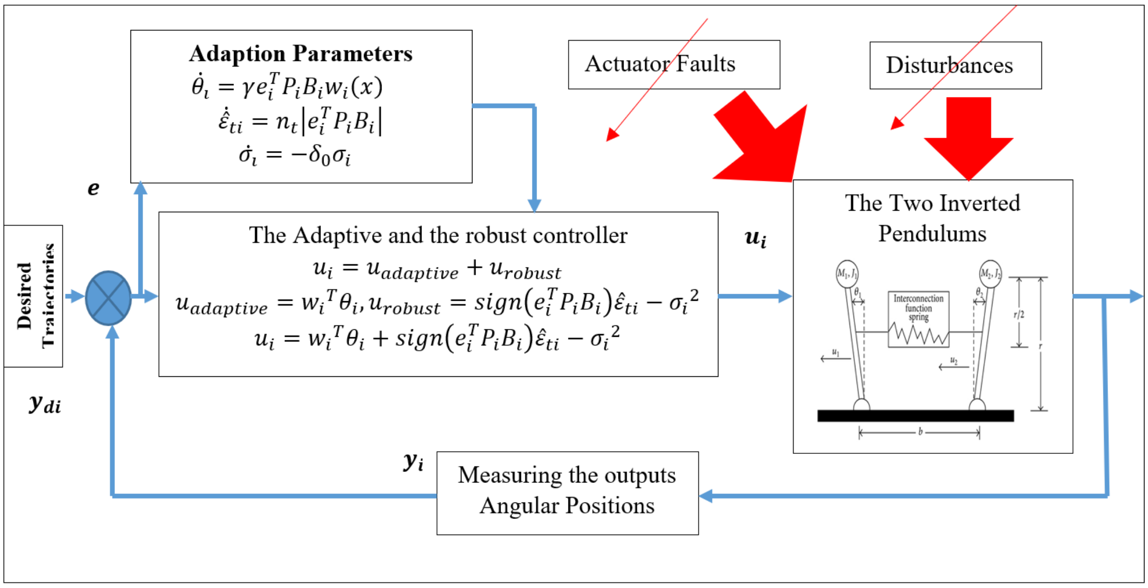

Fig. 6. The overall scheme

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