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Design of Hybrid Controller using Qualitative Simulation Internal Modeling for Inverted Pendulum

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ABSTRACT

Multiple model methods for nonlinear dynamical system control are appealing because local models can be simple and obvious, and global dynamics can be studied in terms of transitions between small operating zones. In this study, we propose that using qualitative models strengthens the multiple model method even more by enabling each local model to explain a huge class of effective nonlinear dynamical systems. Furthermore, reasoning using qualitative models reveals weak necessary conditions sufficient to verify qualitative features like stability analysis. The authors show the method by creating a global controller for the free pendulum. In addition, local controllers are specified and validated by comparing their patterns to basic general qualitative models. Our proposed procedure establishes qualitative limitations on controller designs that are sufficient to ensure the necessary local attributes and to establish feasible transitions between local areas for the existing problems. As a result, the continuous phase picture may be reduced to a simple transitional graph. The degrees of freedom in the system that are not bound by the qualitative description are still accessible to the designer for optimization for any other purpose. An example of a pendulum plant illustrates the effectiveness of the proposed method.

1. Introduction

As a nonlinear, unstable, and high order plant, the inverted pendulum (IP) system [1] has grown in popularity as a benchmark model for assessing the effectiveness of control strategies. There are a large number of nonlinear control issues involving the original control of an IP [2], [3], [4], [5], [6]. An IP is often modeled as a free-rotating pendulum rod is connected to a compact cart that typically moves along a directing bar. It has made a name for itself as a typical mechanical control system for treating, researching, and demonstrating many elements of stability and control theories in scientific literature. In fact, because of its complex dynamics, well-known feedback systems like robots, satellites, rockets, and aircraft may be impressively modeled [7].

Additionally, the control of the rotating inverted pendulum is important in Segway systems, maritime systems, and vibration eliminating devices [8], [9]. Due to several inherent characteristics that make it a highly nonlinear, unstable, underactuated, and multivariable fourth-order system, the analysis and control design of these systems is a difficult challenge. Despite the challenges these systems provide, various studies have been done to stabilize and manage IP networks and several controls including PID, fuzzy logic, and neural networks, have already been introduced in some earlier studies [10], [11], [12], [13], [14]. For instance, in [15] authors studied experimentally and numerically about an adaptive swing-up based sliding mode control technique and its use for a genuine inverted pendulum system. In another research an event-triggered fuzzy controller for an inverted pendulum system was created by authors [16]. We can classify IP controlling and analysing approaches into following:

- The intelligent controller [17], [18], [19].
- The nonlinear trajectory following technique [20], [21].
- The energy-based controller [22], [23], [24], [25].

It is a proven fact that there are an a plenty of nonlinear control problems with the preliminary control of an IP [26], [27], [28]. This model's control challenge has been resolved in a variety of ways. A traditional approach, however, would mostly involve controlling the dynamic force acting on the cart to ensure that the pendulum is balanced at a particular position, such as vertical and at a 90-degree angle [29]. This is a control issue with a single input variable and several output variables. The force on the cart is represented by one input variable. The location of the cart and the skewed angle of rotation between the pendulum rod and a typical direction (often vertical) are two outputs. To achieve the pendulum's balance and bring the system to a stable state, these output variables must be properly regulated. The IP control problem might not be effectively resolved by conventional control methods. The alternative way to solve IP control problems is to use multiple model approach.

Multiple model approaches to the control of complex dynamical systems are attractive because the local models can be simple and intuitive, and global behavior can be analyzed in terms of transitions among local operating regimes [30]. In this paper, we argue that the use of qualitative models further improves the strengths of the multiple model approach by allowing each local model to describe a large class of useful non-linear dynamical systems [31]. In addition, reasoning with qualitative models naturally identifies weak sufficient conditions adequate to prove qualitative properties such as stability. Since a qualitative model only constrains certain aspects of a real system, the remaining degrees of freedom are available for optimization according to any criterion the designer chooses.

Authors implement the Qualitative Simulation Internal Modeling (QSIM) framework for representing qualitative differential equations (QDEs) and doing qualitative simulation to predict the set of all possible behaviors of a QDE and initial state [31]. A QDE is a qualitative abstraction of a set of ODEs, in which the domain of each variable is described in terms of a finite, totally ordered set of landmark values, and an unknown function may be described in terms of regions of monotonic behavior and tuples of corresponding landmark values it passes through. Qualitative simulation predicts a transition graph of qualitative states guaranteed to describe all solutions to all ODE models consistent with the given QDE. By querying QSIM output with a temporal logic model-checker, we can prove universal statements in temporal logic as theorems about sets of dynamical systems described by the QDE [32].

Because it is consistent with nonlinear models, a simple and intuitive QDE model can cover a larger region of the state space than would be possible for a linear ODE. Because a QDE model can express incomplete knowledge, it can be formulated even when the model is not fully specified, and it can express sufficient conditions for a desired guarantee while leaving other degrees of freedom unspecified. These properties are helpful in abstracting the continuous state space of the system to a compact and useful transition graph. The main contributions are given below:

- Firstly, authors define the new discrete mathematical model for highly nonlinear model for hybrid control.
- Secondly, use the *QSIM* technique to estimate the states and stabilize the sliding mode plant.
- Finally, in the simulation section, authors present the pendulum example to show the effectiveness of proposed Algorithm.

2. Problem Statement

2.1. Abstraction from Continuous to Discrete States

Since the discrete transition-graph model allows the analyst to concentrate on the system's large-granularity state rather than its intricate dynamics, it is crucial for thinking about large-scale hybrid systems. The depiction makes it easier to analyze the system using automata theory and temporal logic [33], as well as to create hierarchical representations for understanding dynamics [34].

We divide the state space into a number of areas with distinct interiors, though common border points are possible. Each region-specific dynamical system description must be far shorter than the global system description in order to be helpful. The transition-graph model is then used to abstract each area to a node. The transition from one node to another, which reflects the presence of a pathway between the respective areas across their shared border in the continuous state space, provides the fuzzy basis functions. Think about the collection of continuous paths in the region with beginning states. The abstracted node has no outbound transitions if all of those trajectories remain within the area. The abstracted model contains transitions to each of the associated nodes if certain trajectories cross a zone's boundary and enter another region.

Given a QDE model and a qualitative description of the system's starting state, QSIM predicts all potential behavior for the system. Therefore, qualitative simulation can infer the necessary transitions if the region can be represented qualitatively and the dynamical system constrained to that region can be described by a QDE. A general nonlinear and heterogeneous system's attributes cannot be proved by qualitative modeling and simulation alone. It does, however, offer a far more expressive vocabulary for defining the qualitative and semi-quantitative characteristics of classes of non-linear dynamical systems, as well as deriving characteristics of sets of all potential behaviors of those systems. It gives a designer more freedom and authority to describe the desired characteristics of a dynamical system. Additionally, it offers resources for demonstrating the success of a qualitatively stated design.

2.2. Example: The Free Pendulum

The free pendulum (Fig. 1) is a non-linear dynamical system that is straightforward but not uncomplicated. A common control textbook exercise is to balance a pendulum in the upright position. Machine learning techniques that learn dynamical control rules utilize this job as their aim. The inverted pendulum is a crucial real-world model for a variety of operations, including robot walking and missile launching.By creating a global controller for the free pendulum, we put our strategy on display. By comparing the architecture of local controllers to straightforward general qualitative models, we specify and validate them. By substituting monotonic functions for linear terms, the qualitative framework of QSIM enables us to expand well-known straightforward systems like the damped harmonic oscillator (damped spring). It is simple to demonstrate the relevant qualitative features of the damped spring and significant variations like the spring with negative damping, either using QSIM or analytically (as we do in this study).

The collection of local models that can be included in a heterogeneous hybrid model and have desired attributes is open-ended. Here, we examine several straightforward yet helpful instances. Additionally open-ended at the moment, the collection of relevant transitions between local models may ultimately prove to be finite, at least under qualitative description. Here, we give some helpful examples, but we haven't yet made any recommendations on the set's boundaries. Through this method, the controller designs are given qualitative limitations sufficient to provide the necessary local features and identify potential transitions between local areas. As a result, the continuous phase picture may be reduced to a straightforward transition graph.

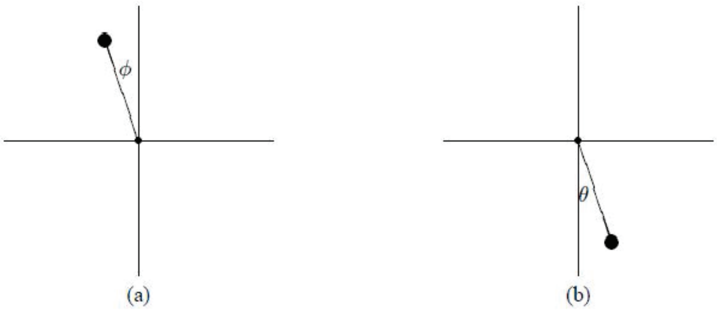

Fig. 1. Local models of the pendulum (where $\phi = \theta - pi$): (a) $\phi = 0$ at the unstable fixed-point, and (b) $\phi = 0$ at the stable fixed-point.

3. Qualitative Properties of Damped Oscillators

Before addressing the pendulum, we need to prove a couple of useful lemmas about the properties of two generic qualitative models: the spring with damping friction and the spring with negative damping. Consider the familiar mass-spring system. The key fact about springs is Hooke's Law, which says that the restoring force exerted by a spring is proportional to its displacement from its rest position. If x represents the spring displacement from rest, then

$$F = ma = m\ddot{x} = -k_1 x$$

We add a damping friction force to the linear model by adding a term proportional to $\dot{x}$ and opposite in direction (Real damping friction is often non-linear).

$$F = ma = m\ddot{x} = -k_1 x - k_2 \dot{x}$$

Rearranging and renaming the constants, we get a linear model of the damped spring:

$$\ddot{x} + b\dot{x} + cx = 0 \quad (1)$$

The linear model is easy to solve, but it embodies simplifying assumptions that are often unrealistic. By generalizing linear terms in equation (1) to monotonic functions, and allowing the functions to be described qualitatively rather than specified precisely, we get a model

$$\ddot{x} + f(\dot{x}) + g(x) = 0$$

that encompasses a large number of precise ODE models, including ones that are much more realistic descriptions of the world.

To make qualitative simulation possible, we must restrict our attention to reasonable functions, which are defined below along with some useful concepts for expressing qualitative models.

Definition 1 *where* $[a, b] \subseteq \Re^*$*, the function* $f: [a, b] \to \Re^*$ *is a reasonable function over* $[a, b]$ *if*
1. f *is continuous on* $[a, b]$*,*
2. f *is continuously differentiable on* (a, b)*,*
3. f *has only finitely many critical points in any bounded interval,*
4. The one-sided limits $Lim_{t\to a^+}\acute{f}(t)$ *and* $Lim_{t\to b^-}\acute{f}(t)$ *exist in* $\Re^*$*. Define* $\acute{f}(a)$ *and* $\acute{f}(b)$ *to be equal to these limits.*

Definition 2 *M+ is the set of reasonable functions* $f : [a, b] \to R^*$ *such that* $\acute{f} > 0$ *over (a, b).*

Definition 3 M_0^+ *is the set of* $f\epsilon M^+$ *such that* $f(0) = 0$*.*

Definition 4 $[x]_0 = sign(x) \in \{+.0, -\}$.

Here we establish the important qualitative properties of the monotonic damped spring model.
Lemma 1 Let $A \subseteq \Re^2$ include (0,0) in its interior, and let S be a system governed by the QDE

$$\ddot{x} + f(\dot{x}) + g(x) = 0 \tag{2}$$

for every $(x, \dot{x}) \in A$, where f and g are reasonable functions such that $f \in M_0^+$ and $[g(x)]_0 = [x]_0$. Then for any trajectory $(x(t), \dot{x(t)})$ of S that lies entirely within A,

$$lim_{t\to\infty}(x(t), \dot{x}(t)) = (0, 0)$$

Proof We rewrite equation (2) as

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_{x2}) = x_2 \\ \dot{x}_2 &= f_2(x_1, x_{x2}) = -f(x_2) - g(x_1) \end{aligned} \tag{3}$$

Because g is a reasonable function, we know that $\acute{g}(0)$ is defined. Since $[g(x)]_0 = [x]_0$, we conclude that $g(0) = 0$ and $\acute{g}(0) > 0$. Any fixed-point of equation (3) must satisfy $\dot{x}_1 = \dot{x}_2 = 0$, which implies that the only fixed point is at $x1 = x2 = 0$.

By the stable manifold theorem [35], the qualitative behavior of the nonlinear system (3) around the fixed point at (0, 0) is the same as that of its local linearization:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\acute{f}(0)x_2 - \acute{g}(0)x_1 \end{aligned} \tag{4}$$

The eigenvalues of (4) are

$$\lambda_{1,2} = 1/2[-\acute{f}(0) \pm \sqrt{\acute{f}(0)^2 - 4\acute{g}(0)}]$$

Because $\acute{f}(0), \acute{g}(0) > 0$ the eigenvalues have negative real parts, so (0, 0) is an asymptotically stable fixed point. When the friction force term $\acute{f}$ is small relative to the spring force term $\acute{g}$, the eigenvalues will be complex, in which case (0, 0) will be a spiral attractor.

Because $[g(x)]_0 = [x]_0$ the spring force is always a restoring force, so we can define a Lyapunov function

$$V(x, \dot{x}) = 1/2\dot{x}^2 + \int_0^x g(x)dx \tag{5}$$

and show that $V(x, \dot{x}) \geq 0$ and that $V = 0$ only at (0,0), and that $d/dtV \leq 0$. This means that S is asymptotically stable at (0,0), and that A can contain no limit cycles.

Together, this tells us that any trajectory $(x(t), \dot{x}(t))$ that enters A eventually terminates at (0,0), for any reasonable functions f and g such that $f \in M_0^+$ and $[g(x)]_0 = [x]_0$. Now we establish similar properties for another monotonic generalization of the damped spring but with negative damping.

Lemma 2 Let $A \subseteq \Re^2$ include (0,0) in its interior, and let S be a system governed by the QDE

$$\ddot{x} - f(\dot{x}) + g(x) \quad = \quad 0 \tag{6}$$

for every $(x, \dot{x}) \in A$, where f and g are reasonable functions such that $f \in M_0^+$ and $[g(x)]_0 = [x]_0$. Then (0,0) is the only fixed point of S in A, and it is unstable. Furthermore, A cannot contain a limit cycle.

Proof The proof of this Lemma is very similar to the previous one. We rewrite equation (6) as

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_{x2}) = x_2 \\ \dot{x}_2 &= f_2(x_1, x_{x2}) = f(x_2) - g(x_1) \end{aligned} \tag{7}$$

As before, because g is a reasonable function, we know that $\acute{g}(0)$ is defined. Since $[g(x)]_0 = [x]_0$, we conclude that g(0) = 0 and $\acute{g}(0) > 0$.

Any fixed-point of equation (7) must satisfy $\dot{x}_1 = \dot{x}_2 = 0$, so the only fixed point is at $x_1 = x_2 = 0$. As before, the qualitative behavior of the nonlinear system (7) around the fixed point at (0, 0) is the same as that of its local linearization:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \acute{f}(0)x_2 - \acute{g}(0)x_1 \end{aligned} \tag{8}$$

The eigenvalues of (8) are

$$\lambda_{1,2} \quad = \quad 1/2[\acute{f}(0) \pm \sqrt{\acute{f}(0)^2 - 4\acute{g}(0)}]$$

In this case, since $\acute{f}(0) > 0$, the eigenvalues have positive real parts, and (0,0) is an unstable fixed point. If the friction force?term $\acute{f}$ is small relative to the spring force term $\acute{g}$, then the eigenvalues will be complex, so (0, 0) will be a spiral repellor.

By the Bendixon negative criterion [35], there can be no periodic orbits contained in A because

$$\partial f_1/\partial x_1 + \partial f_2/\partial x_2 \quad = \quad \acute{f}(x)_2$$

is always positive over A. That is, A cannot contain a limit cycle.

Therefore, except for the unstable fixed-point at (0, 0) itself, any trajectory $(x(t), \dot{x}(t))$ that starts in A, eventually leaves A, for any reasonable functions f and g such that $f \in M_0^+$ and $[g(x)]_0 = [x]_0$. In this context, like that of Lemma 1, we can interpret $V(x, \dot{x})$ from equation (5) as representing the total energy of the system, but here we can show that energy is increasing steadily except at isolated points.

3.1. Proof by Qualitative Simulation

These Lemmas establishing the properties of the monotonic spring models were proved by hand to make this paper self-contained, and because these models are generic and useful. It is also possible to generate proofs of these and similar statements automatically from the QSIM QDE models. The Guaranteed Coverage Theorem states that every real behavior of every model described by the QDE is predicted by QSIM [36], [31]. Then we can use a temporal logic model-checker to establish whether the predicted behavior tree is a model of a specified statement in temporal logic. For universal statements, the completeness of the model-checker and QSIM Guaranteed Coverage combine to show that a positive response from the model-checker implies that the temporal logic statement is a theorem for all behaviors of all dynamical systems consistent with the given QDE [32]. This method of deriving the necessary lemmas using QSIM makes it possible to generalize this approach to more complex models as in [37].

Remark 1 *The proposed algorithm is very generalized and optimized. Our scheme is not limited to electrical and mechanical systems but is also applicable to the biomedical sciences. In addition to factory automation, grid-connected photovoltaic generation plants, and cascaded H-bridge converters, networked control systems are proving to be very useful in practical industrial systems. Modeling, analyzing, and synthesizing NCSs is one of the challenges posed by the introduction of communication networks. For example, there are network-induced delays, data packet dropouts, limited widths, quantization, etc. Future work will focus on modeling nonlinear NCSs using IT2 fuzzy-model-based systems and considering stability, filtering, and controller design problems.*

4. Mathematical Modeling

4.1. A Controller for the Pendulum

By appealing to the qualitative properties of solutions to these very general models, we can give a simple and natural derivation for a controller for the pendulum, able to pump it up and stabilize it in the inverted position.

4.2. Stabilizing the Inverted Pendulum

The pendulum is a mass on a rigid, massless rod, attached to a fixed pivot. The variable ϕ measures the angular position counter-clockwise from the vertical (Fig. 1(a)). We consider only $\phi \in (-\Pi/2, +\Pi/2)$. The angular acceleration due to gravity is $ksin\phi$,and there is a small amount of damping friction $-f(\phi)$, where $f \in M_0^+$. A control action $u(\phi, \dot{\phi})$ exerts angular acceleration at the pivot. The resulting model of the pendulum is:

$$\ddot{\phi} + f(\dot{\phi}) - ksin\phi + u(\phi, \dot{\phi}) \quad = \quad 0 \tag{9}$$

Our goal is to design $u(\phi, \dot{\phi})$ so that the system is asymptotically stable at $(\phi, \dot{\phi}) = (0, 0)$. Lemma 1 provides a simple sufficient condition: make the pendulum behave like a monotonic damped spring.We define the controller for the Balance region to be:

$$u(\phi, \dot{\phi}) \quad = \quad g(\phi) \quad \text{such that} \quad [g(\phi) - ksin\phi]_0 = [\phi]_0 \tag{10}$$

Since $ksin\phi$ increases monotonically with ϕ over $\phi \in (-\Pi/2, +\Pi/2)$, $g(\phi)$ must increase at least as fast in order to ensure that $[g(\phi) - ksin\phi]_0 = [\phi]_0$We can get faster convergence by augmenting the natural damping $f(\dot{\phi})$ with a damping term $h(\dot{\phi})$ included in the control law, giving us

$$u(\phi, \dot{\phi}) \quad = \quad g(\phi) + h(\dot{\phi}) \quad \text{where} \quad [g(\phi) - ksin\phi]_0 = [\phi]_0 \quad h \in M_0^+ \tag{11}$$

If there is a bound u_{max} on the control action u, then the limiting angle ϕ_{max} beyond which the controller cannot restore the pendulum to $\phi = 0$ is given by constraint

$$u_{max} \quad = \quad ksin\phi_{max} \tag{12}$$

The maximum velocity $\dot{\phi}_{max}$ that the Balance controller can tolerate at $\phi = 0$ is then determined by the constraint

$$1/2\dot{\phi}_{max}^2 \quad = \quad \int_0^{\phi_{max}} g(\phi) - ksin\phi d\phi \tag{13}$$

which represents the conversion of the kinetic energy of the system (9) at $(0, \dot{\phi}_{max})$ into potential energy at $(\phi_{max}, 0)$. Therefore, we define the region of applicability for the Balance controller by the ellipse

$$\phi^2/\phi_{max}^2 + \dot{\phi}^2/\phi_{m\dot{a}x}^2 \quad \leq \quad 1 \tag{14}$$

Note that the shapes of the non-linear functions g and h are only very weakly constrained. The qualitative constraints in (11) provide weak sufficient conditions guaranteeing the stability of the inverted pendulum controller. However, there is plenty of freedom available to the designer to select the properties of g and h to optimize any desired criterion.

4.3. Pumping Up the Hanging Pendulum

With no input, the stable state of the pendulum is hanging straight down. We use the variable θ to measure the angular position counter-clockwise from straight down (Figure 1(b)). The goal is to pump energy into the pendulum, swinging it progressively higher, until it reaches the region where the inverted pendulum controller can balance it in the upright position.

Angular acceleration due to gravity is $-ksin\theta$. As before, damping friction is $-f(\dot{\theta})$, where $f \in M_0^+$, and the control action exerts an angular acceleration $u(\theta, \dot{\theta}$ at the pivot. The resulting model of our system is:

$$\ddot{\theta} + f(\dot{\theta}) + ksin\theta + u(\theta, \dot{\theta}) \quad = \quad 0 \tag{15}$$

Without control action, since $[sin\eta]_0 = [\theta]_0$ over $-\pi < \theta < \pi$, the model exactly matches the monotonic damped spring model of Lemma 1, so we know that it is asymptotically stable at $(\theta, \dot{\theta}) = (0, 0)$. Unfortunately, this is not where we want it.

Fortunately, Lemma 2 gives us a sufficient condition to transform the stable attractor at (0,0) into an unstable repellor.We define the controller for the Pump region so that the system is modeled by a spring with negative damping, pumping energy into the system. That is, define

$$u(\theta, \dot{\theta}) \quad = \quad -h(\dot{\theta}) \quad such \quad that \quad h - f \in M_0^+ \tag{16}$$

Starting with any perturbation from (0,0), this controller will pump the pendulum to higher and higher swings. Lemma 2 is sufficient to assure us that there are no limit cycles in the region $-\pi < \theta < \pi$ to prevent the trajectory from approaching $\theta = \pi$ so the Balance control law can stabilize it in the inverted position.

4.4. The Spinning Pendulum

The Spin region represents the behavioral mode of the pendulum when it is spinning freely at high speed. In the Spin region, a simple qualitative controller augments the natural friction of the system with additional damping, to slow the system down toward the two other regions.

$$u(\theta, \dot{\theta}) \quad = \quad f_2(\dot{\theta}) \quad such \quad thatf_2 \in M_0^+ \tag{17}$$

4.5. Bounding the Pump and Spin Regions

One might ask whether the Pump controller could be so aggressive that the pendulum would overshoot the Balance region entirely. Even with augmented damping by the Spin controller, it might be possible to get a limit cycle that alternates between the Pump and Spin regions. (While the analogy is not perfect, this is one aspect of how the van der–Pol oscillator works.)

We can avoid this problem by defining a suitable boundary between the Pump and Spin regions, and showing that the Pump and Spin controllers together define a sliding mode controller [38], forcing nearby trajectories to converge to the boundary. A boundary with the desired properties is the separatrix of the same pendulum,

$$\ddot{\theta} + ksin\theta \quad = \quad 0 \tag{18}$$

without damping friction or control action. It turns out that this boundary will lead straight into the heart of the Balance region, which can be observed in Fig. 2. In Fig. 2 has very important for the stability analysis.

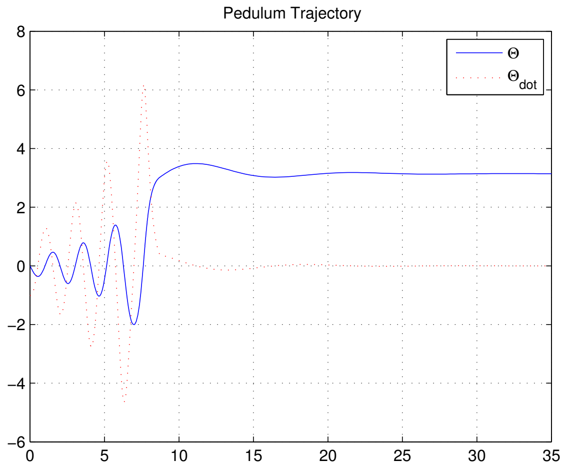

Fig. 2. $\theta(t)$ and $\dot{\theta}(t)$ as the heterogeneous controller pumps a weakly-powered pendulum from $\theta = 0$ to $\theta = \pi$.

A separatrix is a trajectory that starts at an unstable fixed-point of the system and ends at another fixed-point. In the case of the pendulum, the separatrices are the trajectories where the pendulum starts upright and at rest, then swings around once and returns to the upright position, at rest. It is the locus of points $(\theta, \dot{\theta})$ such that the total energy of the system is exactly equal to the potential energy of the motionless pendulum in the upright position.

$$KE + PE \quad = \quad 1/2\dot{\vartheta}^2 + \int_0^{\theta} ksin\theta d\theta = 2k$$

Evaluating the integral and simplifying, we get an equation $s(\theta, \dot{\theta}) = 0$ that defines the separatrix, i.e., the boundary between **Spin** $(s > 0)$ and **Pump** $(s < 0)$.

$$s(\theta, \dot{\theta}) \quad = \quad 1/2\dot{\theta}^2 - k(1 + cos\theta) = 0 \tag{19}$$

We use the method for defining a sliding mode controller from [9] to ensure that trajectories always approach s = 0

Differentiating (19) and substituting for $\ddot{\theta}$, we get

$$\begin{aligned} \dot{s} &= \dot{\theta}\ddot{\theta} + ksin\theta\dot{\theta} \\ &= \dot{\theta}(-f(\dot{\theta}) - ksin\theta - u(\theta, \dot{\theta}) + ksin\theta\dot{\theta} \\ &= -\dot{\theta}f(\dot{\theta}) - \dot{\theta}u(\theta, \dot{\theta}) \end{aligned}$$

Now, examine the Pump region, inside the separatrix where s ¡ 0, and substitute the Pump control law (16) for $u(\theta, \dot{\theta})$.

$$\begin{aligned} \dot{s}_{pump} &= -\dot{\theta}f(\dot{\theta}) + \dot{\theta}h(\dot{\theta}) \quad where \quad h - f \in M_0^+ \\ &= \dot{\theta}(h - f)(\dot{\theta}) \\ &\geq 0 \end{aligned}$$

Similarly, for the Spin region where $s > 0$, substituting its control law (17).

$$\begin{aligned} \dot{s}_{spin} &= -\dot{\theta} f(\dot{\theta}) - \dot{\theta} f_2(\dot{\theta}) \quad f_2 \in M_0^+ \\ &= -\dot{\theta}(f + f_2)(\dot{\theta}) \\ &\leq 0 \end{aligned}$$

This shows that the Pump control law moves the system toward the separator from the inside, and the Spin control law approaches the separator from the outside: the existing control laws define a sliding mode controller with the separator $s = 0$ as the attractor. Once the system gets sufficiently close to the boundary, it will follow the separator, directly into the Balance region. In particular, it is impossible for an aggressive Pump controller to overshoot the Balance region.

5. Numerical Example

5.1. Regions with Fuzzy Boundaries

There are some cases where it would be more convenient to have local models and corresponding regions with gradually changing boundaries rather than sharp ones. A fuzzy set membership function can describe such regions. As a simple example, we can decompose the continuous state space into pure regions, where only one membership function is nonzero, and overlap regions, where two (or perhaps a small number of) regions have nonzero membership functions. Dynamical systems in overlap regions are weighted averages based on membership function values, intersecting local models.

A qualitative *QDE* formalism is particularly useful for representing overlap regions since the shapes of the membership functions are only partially known. Through *QSIM*, it is possible to determine which properties of the local models and the overlapping membership functions guarantee that trajectories through an overlap region can be abstracted as transitions between pure regions.

In Fig. 3, the control law has been presented, while Fig. 4 shows the mode for the inverted pendulum. This was demonstrated by Kuipers and $A^0str - m$ [37] for automatic water tank control and nonlinear chemical reactions. In Fig. 5, the authors demonstrate the superiority of the proposed algorithm over traditional PID control. For better understanding of methodology a block diagram added in Fig. 6.

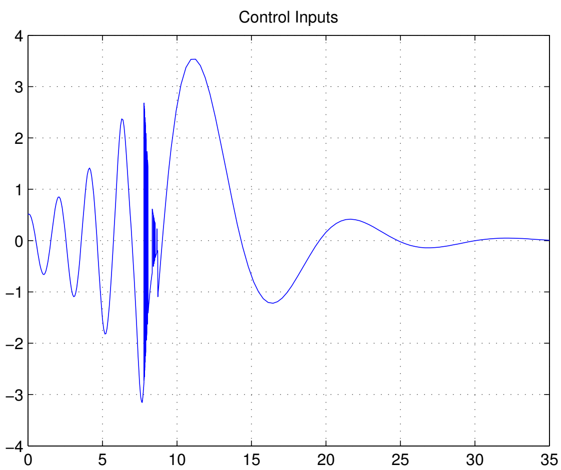

Fig. 3. The control action u(t) shows chattering along the sliding mode.

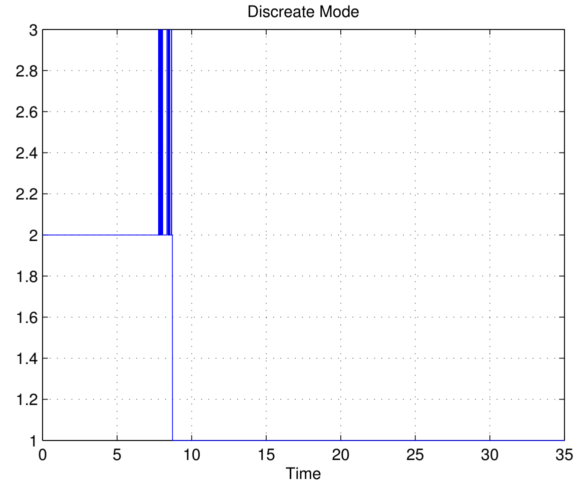

Fig. 4. Discrete mode of Inverted pendulum.

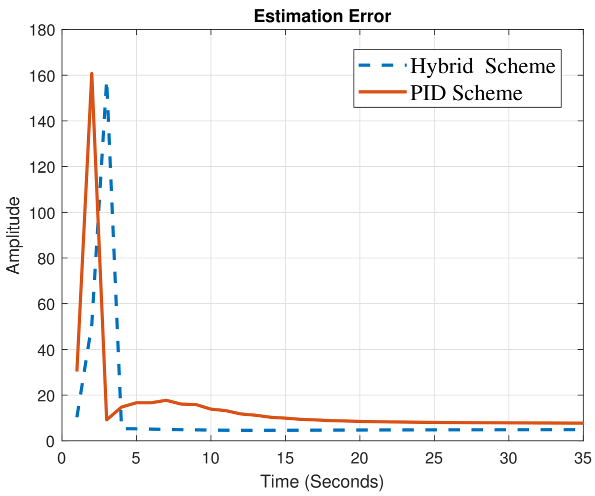

Fig. 5. Estimation error with other schemes mode of Inverted pendulum.

5.2. Feedback Linearization

By adding a term compensating for the nonlinearities in a system, feedback linearization [39] designs a control law that makes the sum linear and therefore suitable for well-understood control methods. Nonlinear systems are difficult to model accurately using this approach. Rather than making the nonlinear system and controller monotonic, we make a much weaker requirement. With bounding envelopes, for example, you can enclose unknown functions even if you don't know the original system. The incompleteness of this form of knowledge will reduce the number of remaining optimization degrees, but will not affect the stability guarantee.

6. Conclusions

In this research, we expressed incomplete knowledge of the dynamics of the uncontrolled plant using qualitative models. In addition, we can separate the essential properties of the controller from the remaining degrees of freedom that can be used for optimization. Natural nonlinear models can

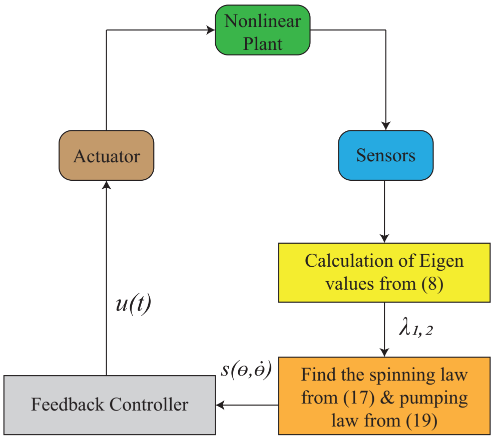

Fig. 6. Block Diagram of proposed algorithm.

also be mathematically expressed, allowing for the use of more natural local models in multi-model frameworks. It is possible to prove the necessary properties of generic qualitative models as well as those of specific qualitative models that describe the controlled system with QSIM. Heterogeneous controllers for free pendulums illustrate these features. This work will be extended to dissipative control and networked sensor systems, where nodes will be working independently for the better utilization of the bandwidth.

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