

Robust Fuzzy Adaptive Control with MRAC Configuration for a Class of Fractional Order Uncertain Linear Systems

Bachir Bourouba ^{a,1}, Samir Ladaci ^{b,c,2,*}, Rachid Illoul ^{d,3}

^a Department of Electrical Engineering, Setif-1 University BP: El Bez, Setif 19000, Algeria

^b Department of E.E.A., National Polytechnic School of Constantine, BP: 75A RP, Ali Mendjeli, Constantine 25000, Algeria

^c SP-Lab Laboratory, University of Mentouri Constantine 1, Department of Electronics, 25000 Constantine, Algeria

^d Department of Automatics, National Polytechnic School of Algiers, (ENP) BP 162, Elharrach, Algiers, 16200, Algeria

¹ bourouba_b@yahoo.fr; ² samir.ladaci@enp-constantine.dz; ³ rachid.illoul@g.enp.edu.dz

* Corresponding Author

ARTICLE INFO

Article History

Received 14 August 2021

Revised 11 September 2021

Accepted 15 September 2021

Keywords

Fractional order system;

Adaptive MRAC control;

Robustness;

Fuzzy logic;

Lyapunov stability

ABSTRACT

This paper investigates a novel robust fractional adaptive control design for a class of fractional-order uncertain linear systems. Based on the Model Reference Adaptive Control (MRAC) configuration, the objective of the proposed controller is to ensure the output of the controlled plant to track the output of a given reference model system, while maintaining the overall closed-loop stability despite external disturbances and model uncertainties. An adaptive fuzzy logic controller is employed to eliminate unknown dynamics and disturbance. Lyapunov stability analysis demonstrates and verifies the desired fractional adaptive control system stability and tracking performance. Numerical simulation results illustrate the efficiency of the proposed adaptive fuzzy control strategy to deal with uncertain and disturbed fractional-order linear systems.

This is an open access article under the [CC-BY-SA](https://creativecommons.org/licenses/by-sa/4.0/) license.



1. Introduction

Fractional calculus is a 300 years old mathematical concept, but no significant impact was achieved in science and engineering until recent years. Recently, considerable attention has been paid to fractional-order systems whose models are described by fractional-order differential equations and especially, to fractional-order control design since it provides more robustness to model uncertainties and better response in comparison with classical controllers. Many works and applications are found in fractional calculus literature [1][2] with many applications such as viscoelastic materials modeling [3], health monitoring [4], modeling and control of robotic systems [5], renewable energy systems [6], chaotic systems [7] and others [8]. One has to mention particularly, that fractional order controllers have been extensively used in many applications to achieve robust performance of the systems [9][10][11].

One of the main research topics in the literature on control theory and engineering for un-

certain linear systems is adaptive control. One can refer to the papers [12][13][14] and the books [15][16][17] and references therein. Design of direct Model Reference Adaptive Control (MRAC) was the subject for many research efforts for linear plants with structured parametric uncertainties. In this control approach, the principle objective is to obtain a system that follows closely the behavior of a system called reference model [18][19].

Fractional order adaptive control is one of the most important emerging research topics in the present century [20][21][22]. Among the proposed fractional adaptive control structures, MRAC-based approaches have attracted many researchers and designers because of its simplicity and efficiency [23]. However, when confronted to model uncertainties and disturbances, other control techniques have to be considered [24][25][26].

An efficient solution is to combine fuzzy systems and estimators to the controller. Fractional order control has been integrated with fuzzy logic in many successful work in order to improve the control system behavior and robustness as in Fractional order PID control [27], fractional adaptive sliding mode control [28][29] and fractional adaptive control [30] of uncertain systems.

Based on the results reported in literature, we propose a novel method to control a class of uncertain linear fractional-order systems using the MRAC configuration. The controller is developed to ensure perfect tracking of the reference model behavior despite model uncertainties and additive disturbances. Numerical simulations show the efficiency of the proposed control scheme.

This paper is organized as follows: Mathematical preliminaries definitions are presented in Section 2. Description of the fuzzy logic system is given in Section 3 while the proposed fractional order model reference adaptive control design and stability proof are developed in Section 4. A numerical simulation example illustrates the efficiency of the method in Section 5 and finally concluding remarks are given in Section 6.

2. Preliminaries on Fractional Order Systems

2.1. Fractional order operators

Fractional order calculus opens a new perspective on integrals and derivatives notion and can be considered as a generalization of integer-order calculus. Let us define the fractional order operator ${}_a D_t^\gamma$, where a denotes the lower limit and t is the upper limit respectively of the operator, and $\gamma \in \mathbb{R}$ is the order of integration or differentiation. There are several definitions for fractional order derivatives. The three most commonly used definitions are the Grünwald-Letnikov, Riemann-Liouville, and Caputo derivative definition [31][32]. if $f(t)$ is a continuous time function, for the order n where $n \in [0, 1]$.

Definition 2.1 The Riemann-Liouville's, fractional order derivative of $f(t)$ of order γ is defined as [33],

$${}^{RL}D_t^\gamma f(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int_{t_0}^t (t-\tau)^{n-\gamma-1} f(\tau) d\tau \quad (1)$$

Where $(n-1) < \gamma < n$, $t \geq t_0$ and $\Gamma(\cdot)$ is the Euler function defined as $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$

Definition 2.2 The Caputo fractional order derivative of the function $f(t)$ of order γ is defined as,

$${}_t^C D_t^\gamma f(t) = \frac{1}{\Gamma(n-\gamma)} \int_{t_0}^t \frac{f^n(\tau)}{(t-\tau)^{\gamma-n+1}} d\tau \quad (2)$$

Definition 2.3 The Grünwald-Letnikov fractional order derivative of the function $f(t)$ of order

$\gamma > 0$ is defined as,

$${}^{GL}_{t_0} D_t^\gamma f(t) = \lim_{h \rightarrow 0} h^{-\gamma} \sum_{j=0}^k (-1)^j \binom{\gamma}{j} f(kh - jh) \quad (3)$$

Where h is the sampling period and $\omega_j^{(\gamma)}$ are the coefficients of the polynomial:

$$(1 - z)^\gamma = \sum_{j=0}^{\infty} (-1)^j \binom{\gamma}{j} z^j = \sum_{j=0}^{\infty} \omega_j^{(\gamma)} z^j \quad (4)$$

The coefficients can be expressed as:

$$\omega_j^{(\gamma)} = \binom{\gamma}{j} = \frac{\Gamma(\gamma + 1)}{\Gamma(j + 1)\Gamma(\gamma - j + 1)}$$

with

$$\omega_0^{(\gamma)} = \binom{\gamma}{0} = 1$$

$$\text{and } k = \left\lceil \frac{t - t_0}{h} \right\rceil.$$

The Grünwald-Letnikov definition of the fractional-order derivative is used in this paper due to its well-understood physical interpretation and implementation easiness.

Remark 1 For the sake of simplicity, we will also note $D_t^\alpha f(t)$ as $f^{(\alpha)}(t)$.

2.2. Some properties of the fractional order derivative

Two general properties of the fractional-order derivative will be used are recalled below:

property 2.4 The additive index law:

$$D_t^\alpha D_t^\beta f(t) = D_t^\beta D_t^\alpha f(t) = D_t^{\alpha+\beta} f(t) \quad (5)$$

where α and β are real numbers.

property 2.5 The Caputo fractional order derivative is a linear operator:

$$D_t^\alpha (af(t) + bg(t)) = aD_t^\alpha (f(t)) + bD_t^\alpha (g(t)) \quad (6)$$

where a and b are real constants.

2.3. Stability of fractional order systems

We recall here some important results on the stability analysis of fractional order systems [34].

Theorem 2.6 Let $x = 0$ be an equilibrium point for the non autonomous fractional order system given by,

$${}^C_{t_0} D_t^\gamma x(t) = h(x(t), t), \quad \gamma \in]0, 1] \quad (7)$$

Let us assume that there exists a continuous function $V(x(t), t)$ such that:

- $V(x(t), t)$ is positive definite.

- ${}^C D_t^\beta V(x(t), t)$, $\beta \in]0, 1]$, is negative semidefinite.

Then the origin of system (7) is Lyapunov stable.

- Furthermore, if $V(x(t), t)$ is decreasing, then the origin of system (7) is Lyapunov uniformly stable.

Lemma 2.7 Assume $h(t) \in \mathfrak{R}$ be a continuous and derivable function. Then, for any time instant $t \geq t_0$, the following inequality holds [34],

$$\frac{1}{2} {}^C D_t^\gamma f^2(t) \leq f(t) {}^C D_t^\gamma f(t) \quad (8)$$

3. Description of the Fuzzy Logic System

Fuzzy Control is the field of control theory based on Fuzzy Set Theory introduced by Zadeh in 1965 [35]. A Fuzzy Logic System (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine using a set of fuzzy rules and the defuzzifier. Usually, a FLS is modeled by the knowledge base of the FLS which is constituted of a collection of fuzzy If-then rules of the following form [28][36]:

R^j : IF x_1 is F_1^j and x_2 is F_2^j and ... and x_n is F_n^j , THEN: y is ϕ^j , $j = 1, 2, \dots, N$.

where $x = (x_1, \dots, x_n)^T$ and y are the fuzzy logic system input and output, respectively. Fuzzy sets F_i^j and ϕ^j , associated with the fuzzy functions $\mu_{F_i^j}(x_i)$ and $\mu_{\phi^j}(y)$ respectively. N is the number of rules. Through singleton function, center average defuzzification, and product inference, the fuzzy logic system can be expressed as [37]:

$$\xi_j = \frac{\prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{j=1}^N \left(\prod_{i=1}^n \mu_{F_i^j}(x_i) \right)} \quad (9)$$

and,

$$\hat{f}(x) = \prod_{j=1}^N \theta_j \xi_j \quad (10)$$

Where $\theta = [\theta_1, \theta_2, \dots, \theta_N]^T = [\theta_1, \theta_2, \dots, \theta_N]^T$ is a vector of the adjustable factors of the consequence part of the fuzzy rule and $\xi = [\xi_1(x), \xi_2(x), \dots, \xi_N(x)]^T$ is a regressive vector with the regressors (fuzzy basis functions) $\xi_j(x)$. We recall this important result [37],

Theorem 3.1 For any real function $f(x)$ continue on a given compact $U \in \mathfrak{R}$, there is a fuzzy system $\hat{f}(x)$ in the form (10) such that:

$$\sup_{x \in U} \|f(x) - \hat{f}(x)\| \leq \epsilon \quad (11)$$

where $\epsilon > 0$ is an arbitrary constant.

4. Fractional Order Model Reference Adaptive Control Design

The MRAC control structure is very efficient for plants with parameters that are unknown or varying slowly in the time, with better performance and robustness than fixed controllers. As illustrated in the block diagram of Fig. 1, it is structured in four main parts; the plant, the controller, the reference model and the adjustment mechanism. In order to introduce the fractional

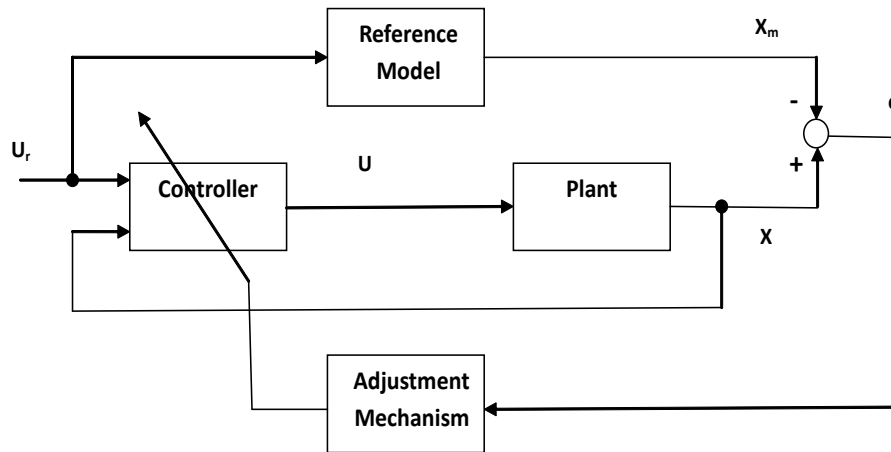


Fig 1. Model reference adaptive control block diagram.

order model reference adaptive control problem, we first consider the class of fractional-order uncertain linear system described by:

$$D_t^\alpha x(t) = A[x(t) + \delta x(t)] + Bu(t) \quad (12)$$

where $x(t) \in \mathfrak{R}^n$ is a state vector and $u(t) \in \mathfrak{R}$ is the system input, $A \in \mathfrak{R}^n \times \mathfrak{R}^n$ and $B \in \mathfrak{R}^n$ have unknown constant parameters. $\delta x(t)$ is regarded as an uncertain dependent nonlinear state perturbation.

In order to introduce the proposed control input, the following assumptions are supposed true:

Assumption 4.1 *There exists a vector $\phi \in \mathfrak{R}^n$, a real function $\omega(t)$ and scalar $\eta \in \mathfrak{R}$ such that:*

$$A\phi = \eta B.$$

and

$$\delta x(t)\phi = B\omega(t)$$

Assumption 4.2 *The states of the system $x(t)$ are measurable.*

Assumption 4.3 *$\delta x(t)$ is unknown but bounded.*

According to Assumption 4.1, the system given by (12) can be written as,

$$D_t^\alpha x(t) = Ax(t) + Bu(t) + B\omega(t) \quad (13)$$

The fractional order reference model was chosen in order to generate the desired trajectory $x_m(t)$ which the plant output has to follow. The reference model is:

$$D_t^\alpha x_m(t) = A_m x_m(t) + B_m u_r(t) \quad (14)$$

The reference model is evidently stable and the input signal $u_r(t) \in \mathfrak{R}$ is a bounded piecewise continuous function. $A_m \in \mathfrak{R}^n \times \mathfrak{R}^n$ and $B_m \in \mathfrak{R}^n$ are known, $x_m(t) \in \mathfrak{R}^n$ is the reference model state vector available at each time t .

The design objective is to make the tracking error $e(t) = x(t) - x_m(t)$ converge to 0 and based on the universal approximation theorem [38], the above fuzzy logic system is capable of uniformly approximating any well-defined nonlinear function over a compact set U_c to any degree of accuracy.

Firstly, the ideal controller is defined as,

$$u(t) = u_a^*(t) + u_f^*(t) \quad (15)$$

where

$$\begin{cases} u_a^*(t) = K_x^{*T} x(t) + K_r^* u_r(t) \\ u_f^*(t) = -\theta^{*T} \xi(t) \end{cases} \quad (16)$$

Where the vectors K_x^* and K_r^* are constant but unknown ideal gains and $u_f^*(t)$ is approximation of smooth function $\omega(t)$.

Comparing the ideal closed-loop transfer function of the controlled plant to the desired reference model, the model-matching conditions are,

$$\begin{cases} A + BK_x^{*T} = A_m \\ BK_r^* = B_m \\ \omega(t) = \theta^{*T} \xi(t) \end{cases} \quad (17)$$

In the adaptive control problem, the parameters of A and B are unknown and so, the control (15) cannot be implemented. Therefore, an adaptive controller which has the same structure as in (15) is used as follows,

$$u(t) = K_x^T(t)x(t) + K_r(t)u_r(t) + u_f(t) \quad (18)$$

Where $K_x(t)$ and $K_r(t)$ are the estimates of K_x^* and K_r^* at time t , and we search an adaptive law to generate $K_x(t)$ and $K_r(t)$ on-line with,

$$u_f(t) = -\theta^T \xi(t) \quad (19)$$

In order to develop the adaptive laws for $K_x(t)$, $K_r(t)$ and $\theta(t)$, we need an error equation in terms of tracking error $e(t) = x(t) - x_m(t)$ and the parameters errors

$$\begin{cases} \tilde{K}_x(t) = K_x(t) - K_x^* \\ \tilde{K}_r(t) = K_r(t) - K_r^* \\ \tilde{\theta}(t) = \theta(t) - \theta^* \end{cases} \quad (20)$$

Based on the certainty equivalence principle and the proposed adaptive control input (18), the closed-loop fractional order error dynamic system can be described by,

$$D_t^\alpha e(t) = A_m e(t) + B [\tilde{K}_x^T(t)x(t) + \tilde{K}_r(t)u_r(t) - \tilde{\theta}^T(t)\xi(t)] \quad (21)$$

or

$$D_t^\alpha e(t) = A_m e(t) + B_m \left[\frac{\tilde{K}_x^T(t)x(t)}{K_r^*} + \frac{\tilde{K}_r(t)u_r(t)}{K_r^*} - \frac{\tilde{\theta}^T(t)\xi(t)}{K_r^*} \right] \quad (22)$$

Let us define the matrix $P = P^T \in \Re^n \times \Re^n$ solution of the following Lyapunov equation:

$$PA_m + A_m^T P = -Q \quad (23)$$

for any chosen matrix $Q \in \Re^n \times \Re^n$ such that $Q = Q^T$. Then we present the main result of this paper,

Theorem 4.4 Consider the fractional-order uncertain linear system (12) in the presence of the uncertainties subject to assumptions 4.1-4.3. The robust adaptive controller defined by (18)-(20) with adaptation laws given by

$$\begin{cases} D_t^\alpha K_x(t) = -\text{sign}(K_r^*) \Gamma_x x(t) e^T(t) P B_m \\ D_t^\alpha K_r(t) = -\text{sign}(K_r^*) \gamma_r u_r(t) e^T(t) P B_m \\ D_t^\alpha \theta(t) = \text{sign}(K_r^*) \Gamma_\theta \xi(x) e^T(t) P B_m \end{cases} \quad (24)$$

where the positive-definite matrices $\Gamma_x = \Gamma_x^T \in \mathbb{R}^n \times \mathbb{R}^n$ and $\Gamma_\theta = \Gamma_\theta^T \in \mathbb{R}^p \times \mathbb{R}^p$ and the scalar $\gamma_r \in \mathbb{R}^+$ are design parameters. Ensures that all the closed-loop signals are bounded, and the tracking errors converge to zero.

Proof of Theorem 4.4:

In order to analyse the closed-loop stability, let us consider the following Lyapunov candidate function,

$$V = \frac{1}{2} e^T P e + \frac{1}{2 |K_r^*|} \left[\tilde{K}_x^T \Gamma_x^{-1} \tilde{K}_x + \frac{1}{\gamma} \tilde{K}_r^2 + \tilde{\theta}^T \Gamma_\theta^{-1} \tilde{\theta} \right] \quad (25)$$

where Γ_x and Γ_θ are the adaptation rate matrices for K_x and K_θ respectively. γ is as adaptive scalar gain that affects the speed of the parameter adaptation.

Taking the fractional order derivative of (25) with respect to time and using Lemma 2.7, one has

$$\begin{aligned} D_t^\alpha V(t) \leq & \frac{1}{2} e^T P e^{(\alpha)} + \frac{1}{2} e^{(\alpha)T} P e + \dots \\ & + \frac{1}{|K_r^*|} \left[\tilde{K}_x^T \Gamma_x^{-1} \tilde{K}_x^{(\alpha)} + \frac{1}{\gamma} \tilde{K}_r \tilde{K}_r^{(\alpha)} + \tilde{\theta}^T \Gamma_\theta^{-1} \tilde{\theta}^{(\alpha)} \right] \end{aligned} \quad (26)$$

By substituting (22) into (26) we obtain

$$\begin{aligned} V^{(\alpha)} \leq & e^T (P A_m + A_m^T P) e + \dots \\ & + \left[\frac{1}{K_r^*} e^T P B_m \tilde{K}_x x + \frac{1}{|K_r^*|} \tilde{K}_x^T \Gamma_x^{-1} \tilde{K}_x^{(\alpha)} \right] + \dots \\ & + \left[\frac{1}{K_r^*} e^T P B_m \tilde{K}_r u_r + \frac{1}{\gamma |K_r^*|} \tilde{K}_r \tilde{K}_r^{(\alpha)} \right] + \dots \\ & + \left[\frac{1}{K_r^*} e^T P B_m \tilde{\theta} + \frac{1}{|K_r^*|} \tilde{\theta}^T \Gamma_\theta^{-1} \tilde{\theta}^{(\alpha)} \right] \end{aligned} \quad (27)$$

Then, in order to make $V^{(\alpha)} \leq 0$ we choose the adaptive laws for $K_x(t)$, $K_r(t)$ and $\theta(t)$ as specified in (24), namely

$$\begin{cases} D_t^\alpha K_x(t) = -\text{sign}(K_r^*) \Gamma_x x(t) e^T(t) P B_m \\ D_t^\alpha K_r(t) = -\text{sign}(K_r^*) \gamma_r u_r(t) e^T(t) P B_m \\ D_t^\alpha \theta(t) = \text{sign}(K_r^*) \Gamma_\theta \xi(x) e^T(t) P B_m \end{cases}$$

It follows that

$$V^{(\alpha)} \leq -e^T Q e \quad (28)$$

Using Barbalat's lemma [15][39], we conclude that the system is Lyapunov stable with all variables bounded and converge to zero: $e(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$ for all initial conditions of the variables $x(0) \in \mathbb{R}^n$. This completes the proof.

Remark 2 *The proposed adaptive fuzzy controller for the class of uncertain fractional order linear systems has never been designed before to the best of our knowledge. Based on the MRAC configuration, the system is forced to behave like the desired reference model despite the lack of information and uncertainties on the system's signals. Most of existing similar approaches consider nonlinear systems [40].*

5. Simulation Results and Discussion

In this section we will give a numerical example to illustrate the efficiency of the proposed fuzzy robust adaptive scheme. Consider the plant represented by a state-space model of the form (12) with,

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (29)$$

with the disturbance signal modeled as,

$$\omega(t) = 0.8 \cos(0.1 t) \sin(2.5 t) \quad (30)$$

and consider the reference model represented with state-space equation (14) with,

$$A_m = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ 4 \end{bmatrix} \quad (31)$$

For the proposed fuzzy adaptive controller design, we chose the following membership functions for error and error derivative,

$$\begin{aligned} \mu_{F_1^j}(e_i) &= \exp\left[-\frac{e_i + 1}{0.75}\right] \\ \mu_{F_2^j}(e_i) &= \exp\left[-\left(\frac{e_i + 1}{0.75}\right)^2\right] \\ \mu_{F_3^j}(e_i) &= \exp\left[-\left(\frac{e_i}{0.75}\right)^2\right] \\ \mu_{F_4^j}(e_i) &= \exp\left[-\left(\frac{e_i - 0.75}{0.75}\right)^2\right] \\ \mu_{F_5^j}(e_i) &= \exp\left[-\left(\frac{e_i - 1.5}{0.75}\right)^2\right] \end{aligned} \quad (32)$$

where $i = 1, 2$.

The simulation test parameter settings are as follows:

$$\Gamma_x = \begin{bmatrix} 100 & 0 \\ 0 & 50 \end{bmatrix}, \quad \Gamma_r = 10, \quad (33)$$

With the initial conditions,

$$x_0 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \quad x_{0m} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (34)$$

Simulation results are shown in Figs. 2-6, from which we can conclude that the stability of the system are guaranteed. Fig. 5 shows that the error signal converges rapidly towards zero. Fig. 6 shows also that all the varying gains are bounded during the control action.

It is also noticeable that error signal $e(t)$ converges relatively slowly to zero, approximately in 30s (response time). This fact can be explained by the learning phase for adaptive control in presence of uncertainties and disturbances. However, the system is able to follow the varying reference perfectly after this limited time.

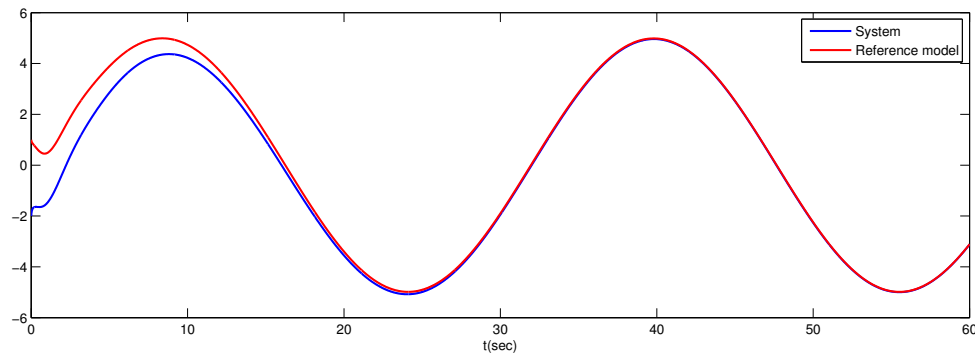
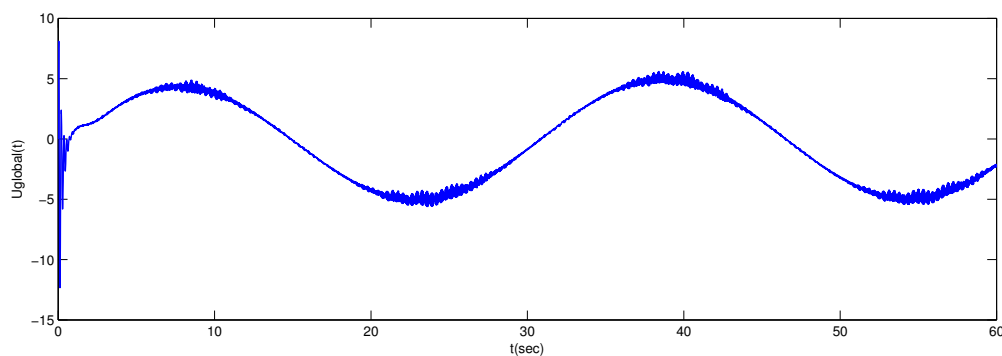


Fig 2. State response.

Fig 3. Control input u .

6. Conclusion

In this paper, we propose a new robust fractional adaptive control design for a class of fractional order uncertain linear systems. We introduce a fractional order adaptive fuzzy controller using the MRAC configuration in order to deal with external disturbances and model uncertainties while forcing the output of the controlled plant to track the output of a given reference model system, while maintaining the overall closed loop stability and signals convergence. The stability analysis is performed using Lyapunov stability theorem. Simulation results illustrate the efficiency of the proposed fuzzy adaptive controller to deal with uncertain fractional order linear systems. Further research will concentrate on application of this control scheme to real processes and compare its performance to other available control techniques in the same operating conditions. Extension to fractional order linear systems with delays is also a challenge.

References

- [1] B. Bourouba, S. Ladaci, and A. Chaabi, "Moth-flame optimisation algorithm-based fractional order PIAD μ controller with MRAC tuning configuration," *International Journal of Systems, Control & Communication*, vol. 9, no. 2, pp. 148–171, 2018. <https://doi.org/10.1504/IJSCC.2018.090745>
- [2] B. Bourouba and S. Ladaci, "A Novel Non-integer Indirect Adaptive Control for Non-integer Order Systems with Non-prior Knowledge," *Engineering, Technology & Applied Science Research*, vol. 10, no. 1, pp. 5186–5190, 2020. <https://doi.org/10.48084/etasr.3236>
- [3] K. A. Lazopoulos, D. Karaoulanis, and A. K. Lazopoulos, "On fractional modelling of viscoelastic mechanical systems," *Mechanics Research Communications*, vol. 98, pp. 54–56, 2019. <https://doi.org/10.1016/j.mechrescom.2017.08.006>
- [4] K. Leyden and B. Goodwine, "Fractional-order system identification for health monitoring," *Nonlin-*

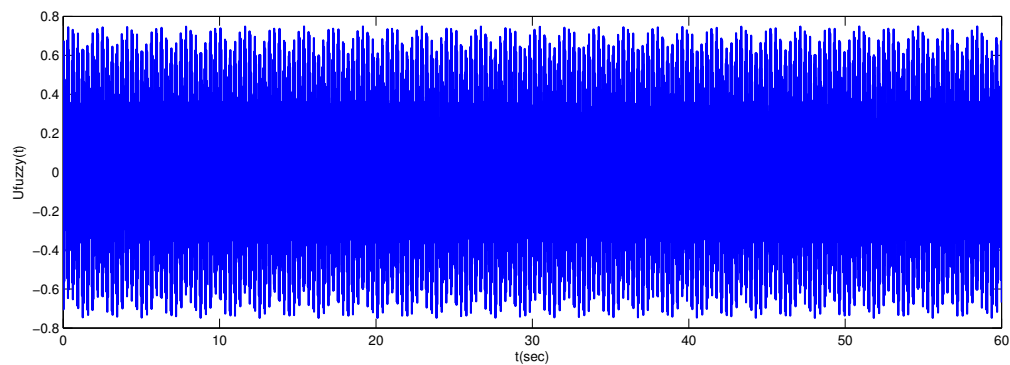


Fig 4. Fuzzy control input u_f .

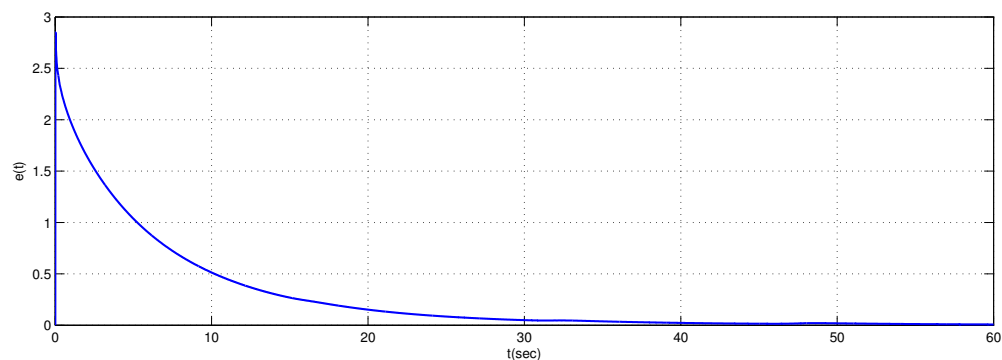


Fig 5. Error signal $e(t)$.

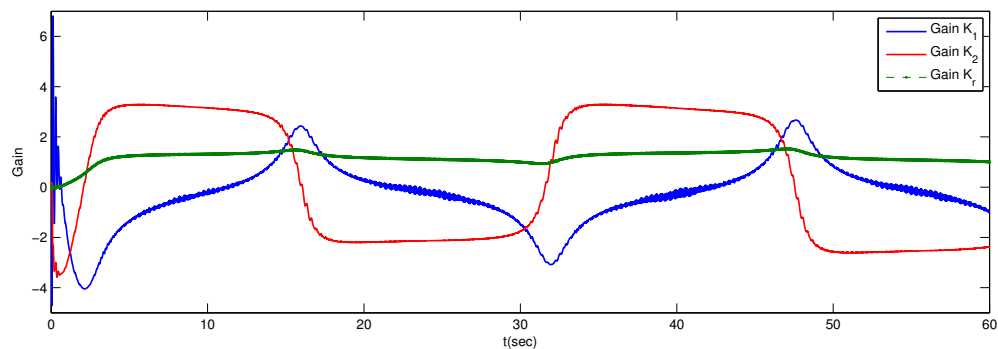


Fig 6. Adaptive gains $K(t)$.

ear Dynamics, vol. 92, no. 3, pp. 1317-1334, 2018. <https://doi.org/10.1007/s11071-018-4128-y>

- [5] M. Gonzalez-Fierro, C. A. Monje, and C. Balaguer, "Fractional control of a humanoid robot reduced model with model disturbances," *Cybernetics and Systems an International Journal*, vol. 47, no. 6, pp. 445-459, 2016. <https://doi.org/10.1080/01969722.2016.1187031>
- [6] S. Djebbari, S. Ladaci, A. Metatla, and H. Balaska, "Fractional-order model reference adaptive control of a multi-source renewable energy system with coupled dc/dc converters power compensation," *Energy Systems*, vol. 11, no. 2, pp. 315-355, 2020. <https://doi.org/10.1007/s12667-018-0317-5>
- [7] M. S. Tavazoei, M. Haeri, S. Jafari, S. Bolouki, and M. Siami, "Some applications of fractional calculus in suppression of chaotic oscillations," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 11, pp. 4094-4101, 2008. <https://doi.org/10.1109/TIE.2008.925774>

-
- [8] S. Ladaci and Y. Bensafia, "Indirect fractional order pole assignment based adaptive control," *Engineering Science and Technology, an International Journal*, vol. 19, no. 1, pp. 518–530, 2016. <https://doi.org/10.1016/j.jestch.2015.09.004>
- [9] W. J. Qiao, X. Q. Tang, S. Q. Zheng, Y. L. Xie, and B. Song, "Adaptive two-degree-of-freedom PI for speed control of permanent magnet synchronous motor based on fractional order GPC," *ISA Transactions*, vol. 64, pp. 303–313, 2016. <https://doi.org/10.1016/j.isatra.2016.06.008>
- [10] L. Angel and J. Viola, "Fractional order PID for tracking control of a parallel robotic manipulator type delta," *ISA Transactions*, vol. 79, pp. 172–188, 2018. <https://doi.org/10.1016/j.isatra.2018.04.010>
- [11] S. Singh, V. K. Tayal, H. P. Singh, and V. K. Yadav, "Fractional Control Design of Renewable Energy Systems," 8th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO), Noida, India, 4–5 June 2020. <https://doi.org/10.1109/ICRITO48877.2020.9197902>
- [12] P. E. Caines and S. Lafortune, "Adaptive control with recursive identification for stochastic linear systems," *IEEE Transactions on Automatic Control*, vol. AC-29, pp. 312–321, 1984. <https://doi.org/10.1109/TAC.1984.1103520>
- [13] F. Ahmed-Zaid, P. Ioannou, K. Gousman, and R. Rooney, "Accommodation of failures in the f-16 aircraft using adaptive control," *IEEE Control Systems Magazine*, vol. 11, no. 1, pp. 73–78, 1991. <https://doi.org/10.1109/37.103360>
- [14] B. M. Mirkin and P. O. Gutman "Model reference adaptive control of state delayed system with actuator failures," *International Journal of Control*, vol. 78, no. 3, pp. 186–195, 2005. <https://doi.org/10.1080/00207170500073814>
- [15] P. Ioannou and J. Sun, "Robust Adaptive Control," New York: Dover Publications Inc., 1996.
- [16] I. D. Landau, R. Lozano, and M. M'Saad, "Adaptive Control," London: Springer-Verlag, 1998. <https://doi.org/10.1007/978-0-85729-343-5>
- [17] G. Tao, "Adaptive Control Design and Analysis," USA: Wiley-IEEE Press, 2003. <https://doi.org/10.1002/0471459100>
- [18] M. Abedini, M. A. Nojournian, H. Salarieh, and A. Meghdari, "Model reference adaptive control in fractional order systems using discretetime approximation methods," *Communications in Nonlinear Science and Numerical Simulation*, vol. 25, no. 1-3, pp. 27–40, 2015. <https://doi.org/10.1016/j.cnsns.2014.11.012>
- [19] J. P. Nelson and M. J. Balas, "Direct model reference adaptive control of linear systems with input/output delays," *Numerical Algebra, Control & Optimization*, vol. 3, no. 3, pp. 445–462, 2013. <https://doi.org/10.3934/naco.2013.3.445>
- [20] B. M. Vinagre, I. Petras, I. Podlubny, and Y. Q. Chen, "Using fractional order adjustment rules and fractional order reference models in model-reference adaptive control," *Nonlinear Dynamics*, vol. 29, no. 1-4, pp. 269–279, 2002. <https://doi.org/10.1023/A:1016504620249>
- [21] S. Ladaci and A. Charef, "On fractional adaptive control," *Nonlinear Dynamics*, vol. 43, no. 4, pp. 365–378, 2006. <https://doi.org/10.1007/s11071-006-0159-x>
- [22] B. Shi, J. Yuan, and C. Dong, "On fractional model reference adaptive control," *The Scientific World Journal*, vol. 2014, p. 521625, 2014. <https://doi.org/10.1155/2014/521625>
- [23] K. Bouziane, A. Djouambi, and S. Ladaci, "Fractional-order model reference adaptive controller design using a modified MIT rule and a feed-forward action for a DC-DC boost converter stabilization," *Proc. the 5th Int. Conf. on Elec. Eng., ICEE'2017*, pp. 1–6. Bumerdes, Algeria: IEEE, Oct. 2017. <https://doi.org/10.1109/ICEE-B.2017.8192132>
- [24] H. Li, L. Bi, and J. Yi, "Sliding-Mode Nonlinear Predictive Control of Brain-Controlled Mobile Robots," *IEEE Transactions on Cybernetics*, pp. 1–13, 2020. <https://doi.org/10.1109/TCYB.2020.3031667>
- [25] S. Roy, S. Baldi, and L. M. Fridman, "On adaptive sliding mode control without a priori bounded uncertainty," *Automatica*, vol. 111, no. 108650, 2020. <https://doi.org/10.1016/j.automatica.2019.108650>
- [26] M. Boudjellal and R. Illoul, "High-order sliding mode and high-gain observers for state estimation and fault reconstruction for a nonlinear CSTR," 2017 6th International Conference on Systems and Control (ICSC), Batna, Algeria, 7–9 May, 2017. <https://doi.org/10.1109/ICoSC.2017.7958678>
-

-
- [27] A. Kumar and V. Kumar, "Design of interval type-2 fractional-order fuzzy logic controller for redundant robot with artificial bee colony," *Arabian Journal for Science and Engineering*, vol. 44, no. 3, pp. 1883–1902, 2019. <https://doi.org/10.1007/s13369-018-3207-1>
- [28] B. Bourouba and S. Ladaci, "Robust fuzzy adaptive sliding mode stabilization for fractional order chaos," *Algorithms*, vol. 11, no. 7, p. 101, 2018. <https://doi.org/10.3390/a11070101>
- [29] K. Khettab, Y. Bensafia, and S. Ladaci, "Enhanced fuzzy adaptive control of uncertain fractional chaotic systems using a fractional sliding mode approach," *Proc. 5th Int. Conf. Elec. Eng. (ICEE-B)*, pp. 1–6. Boumerdes, Algeria: IEEE, 29-31 Oct. 2017. <https://doi.org/10.1109/ICEE-B.2017.8192138>
- [30] K. Khettab, S. Ladaci, and Y. Bensafia, "Fuzzy adaptive control of fractional order chaotic systems with unknown control gain sign using a fractional order Nussbaum gain," *IEEE/CAA Journal of Automatica Sinica*, vol. 6, no. 3, pp. 816–823, 2019. <https://doi.org/10.1109/JAS.2016.7510169>
- [31] M. P. Aghababa, "Stabilization of a class of fractional-order chaotic systems using a non-smooth control methodology," *Nonlinear Dynamics*, vol. 89, no. 8, pp. 1357–1370, 2017. <https://doi.org/10.1007/s11071-017-3520-3>
- [32] B. Bourouba, S. Ladaci, and A. Chaabi, "Reduced-order model approximation of fractional-order systems using differential evolution algorithm," *Journal of Control, Automation and Electrical Systems*, vol. 29, no. 1, pp. 32–43, 2018. <https://doi.org/10.1007/s40313-017-0356-5>
- [33] K. Miller and B. Ross, "An Introduction to the Fractional Calculus and Fractional Differential Equations," Wiley, 1993.
- [34] M. A. Duarte-Mermoud, N. Aguila-Camacho, J. Gallegos, and R. Castro-Linares, "Using general quadratic lyapunov functions to prove lyapunov uniform stability for fractional order systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 22, no. 1-3, pp. 650–659, 2015. <https://doi.org/10.1016/j.cnsns.2014.10.008>
- [35] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, pp. 338–353, 1965. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- [36] J. M. Mendel, "Fuzzy logic systems for engineering: a tutorial," *Proceedings of the IEEE*, vol. 83, pp. 354–377, 1995. <https://doi.org/10.1109/5.364485>
- [37] L.-X. Wang, "Adaptive Fuzzy Systems and Control: Design, and Stability Analysis," Prentice-Hall, 1994. <https://dl.acm.org/doi/abs/10.5555/174457>
- [38] Xiao-Jun Zeng, J. A. Keane, and D. Wang, "Fuzzy systems approach to approximation and stabilization of conventional affine nonlinear systems," *2006 IEEE International Conference on Fuzzy Systems*, 2006, pp. 277–284. <https://doi.org/10.1109/FUZZY.2006.1681726>
- [39] H. Balaska, S. Ladaci, and A. Djouambi, "Direct fractional order MRAC adaptive control design for a class of fractional order commensurate linear systems," *Journal of Control and Decision*, vol. 8, no. 3, pp. 1–15, 2021. <https://doi.org/10.1080/23307706.2020.1809022>
- [40] Y. Sun, Y. Liu, and Ling Li, "Fuzzy Adaptive Control for Fractional Nonlinear Systems with External Disturbances and Unknown Control Directions," *Journal of Mathematics*, vol. 2020, p. 6677704, 2020. <https://doi.org/10.1155/2020/6677704>