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High Gain Observer Based Backstepping Control Design for Nonlinear Single-Axis Driven Systems

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ABSTRACT

In this paper, a backstepping (BS) control design approach is proposed for tracking angular position control problem of a single-input and singleoutput (SISO) nonlinear single-axis driven system. To implement proposed BS control, the states of the system should be available. To address this problem, a high gain observer (HGO) is introduced for estimating the states. The design parameters of the HGO based BS controller have been optimized using the circle search algorithm (CSA). Compare to other optimization algorithm, the CSA explores the search space in a circular trajectory which can enhance local exploitation. The CSA uses integral of absolute error (IAE) as the performance index for the tuning process. The effectiveness of the proposed controller is demonstrated through simulations. Firstly, for observer evaluation, simulation outcomes indicate that the HGO is capable to estimate the states of the system successfully. However, to evaluate the BS with other nonlinear controllers for tracking control problem, the synergetic (SG) control is proposed. The simulated data results based on IAE index revealed that the BS control has lower IAE value than the SG control where the value of the IAE of the system with the BS control is reduced by 19.4% in compares with the system with the SG control.

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1. Introduction

Authors and engineers in research centers and educational institutions have paid great attention to single-axis driven systems as an educational platform in the fields of physics and mechatronics, and even for students in the field of implementing electronic control circuits [1]-[2]. Engineers and researchers have also been interested about the system for several application aspects, as it represents the simplest model for vertical take-off aircraft and autonomous unmanned aerial vehicles UAVs [3]-[4]. Furthermore, its basic structure is based on the concept of an inverted pendulum and a normal pendulum, which expands the scope of applications further and more comprehensively [5]. As the mathematical model describing this system is inherently a nonlinear model, it has become an excellent prototype for conducting numerous research and development activities in the domain of developing control and stability techniques, as well as creating reliable nonlinear controllers [6]-[7].

According to the general description of the single-axis driven system under discussion that based on a simple physical concept of how to balance a lever affected by the weight force of a given





mass at one end and the force of propeller air thrust at the other end, and control the rotational motion of the lever at any desired angle [8]. Such system is distinguished from the inverted pendulum system and the normal pendulum is the possibility of achieving balance and stability at any required angle, using robust and well-designed controllers capable of overcoming the difficulties of nonlinearity in the system [9].

To stabilized angle of the system to any degree and to overcome the inherent instability of the mathematical model of this system, considerable scientific research papers have been published devoted to developing control methods and techniques to improve the accuracy of the performance of system. Charoensuk et al. [10] studied the system with its nonlinear mathematical model and a performance comparison was made between a traditional Proportional-Integral-Derivative (PID) controller and an integral-proportional-plus-derivative (I-PD) controller, where the robustness and speed of the performance of the latter type was confirmed. Ahmed and Al-Khazraji [8] presented a comparative study three controllers including the PID controller, a state feedback controller, and a sliding mode control (SMC). To achieve optimal controller performance, it was necessary to finetune the controller parameters using Gorilla Troops Optimization (GTO). The results showed that the SM controller demonstrated superior transient performance in overcoming the influence of external disturbances. To evaluate the tracking performance, Al-Khazraji [11] examined the performance of SM controller in compares with synergetic (SG) controller. Sparrow search optimization (SSO) was employed to guarantee optimal performance via finding the optimum designing parameters of the each controller. The outcomes revealed that the SG controller exhibits a slightly better performance than SM controller. However, the control law shows a distinct difference in the performance where the SM controller exhibits chattering problem while this phenomenon is not appearing in the SG controller. Adaptive control for single-axis driven system was performed by [2] and [4]. Raffuddin and Khan [12] utilized the data collected from the closed-loop control of a classical real-time PID controller to design a nonlinear controller based on artificial neural networks.

Due to the existence of the parameter uncertainty, the unknown external disturbance and the nonlinearity of the single-axis driven system, designing of a robust tracking control still ongoing challenging task. Regardless the numbers of feedback control schemes that have been proposed to single-axis driven system. Most of these controllers are model-based control methods and the states of the system are required to generate the control law. It was observed that addressing the problem of estimation of the states is not considered. Therefore, this work proposed an observer to estimate the states of the system. The conventional Luenberger observer (LO) was the early technique utilized to estimate the states of linear systems [13]. Later, High-gain observers (HGO) developed by Khalil and Esfandiari for nonlinear systems [14]. To end this, the purpose of this paper is to investigate the ability of the HGO to estimate the states of the single-axis driven system. Then, the performance of designing backstepping (BS) and SG controllers based on the estimation states is examined. To address the problem of finding the best tuning variables of the controller and the observer, the circle search algorithm (CSA) based on integral of absolute error (IAE) is introduced. Comparing to other state-of-art optimization algorithms, CSA have several potential advantages. For example, in contrast to the most of the well-known tuning algorithm which they are struggle with fine-tuning near local optima, CSA explores the search space in a circular trajectory, which can enhance local exploitation.

2. Mathematical Model

The mathematical model of the single-axis driven systems is given in this section. In Fig. 1, the schematic diagram of the single-axis driven system is given. The system have a DC motor drives a propeller attached to pendulum which swing due to gravity in a vertical plane and the propeller can exert torque and generate thrust acts along its axis that affects pendulum motion [11]. The application of such system can be found in robotics and other control applications such as the rotary inverted pendulum or pendulum on drone setups. And the mathematical model of a rotary pendulum

with a propeller at the end of the pendulum includes mechanical dynamics of the pendulum due to gravity, damping, and a torque or thrust from the propeller and electrical dynamics of the DC motor.

To establish the mathematical model of the system, the Newton's second law for rotation is used as follows [2]:

$$J\ddot{\theta} = -\tau_{\rm d} - \tau_{\rm g} + \tau_{\rm m} \tag{1}$$

where J is the inertia of moment, $\ddot{\theta}$ is the angular acceleration, τ_g is the gravitational torque, τ_d is the damping torque, and τ_m is the generates torque due to the attached motor.

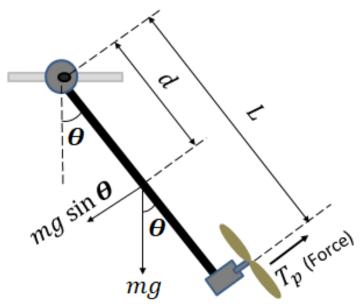


Fig. 1. Single-axis driven system

The system's τ_g , τ_d and τ_m are expressed as follows [4]:

$$\tau_{g} = c\dot{\theta} \tag{2}$$

$$\tau_{\rm d} = {\rm mgd} \sin \theta \tag{3}$$

$$\tau_{\rm m} = K_{\rm m} v \tag{4}$$

where θ , $\dot{\theta}$, c, g, m, K_m, d, and v are the angular position, the angular velocity, the viscous damping coefficient, the acceleration of gravity, the mass of propeller, the distance between the suspending point and the mass center, constant of the DC motor, applied voltage respectively.

Let x_1, x_2 and u refer to $\theta, \dot{\theta}$ and u. The differential equations that represent the dynamic motion of the single-axis driven system are illustrated as follows:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{5}$$

$$\dot{x}_{2} = \frac{-cx_{2}(t) - mgd \sin x_{1} + K_{m}u}{I}$$
 (6)

Eq. (2) can be rewrites as follows:

$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + \mathbf{b}\mathbf{u} \tag{7}$$

where
$$f(x) = \frac{-cx_2(t) - mgd \, sinx_1}{J}$$
 , $b = \frac{K_m}{J}$

3. Controller Design

Feedback controller design is used to improve system's performance [15]-[24]. In many control system such as robotics, the main objective of the control problem is to follow sort of reference signal (i.e. tracking) including step, ramp or/and sinusoidal [25]. In this paper, two nonlinear controllers are developed for tracking angular position control problem of a single-input and single-output (SISO) nonlinear single-axis driven system including a backstepping (BS) control and synergetic (SG) control. To implement proposed controllers, the states of the system should be available. To address this problem, a high gain observer (HGO) is introduced for estimating the states. Therefore, the following subsections explain the details of BS, SG and HGO.

3.1. Backstepping Control

BS control is well-known nonlinear control approach applicable in a number of dynamic systems. Basically, the control law in the BS is formulated recursively and systematic based on the Lyapunov function. In particular, various applications of BS control can be found such as twin-tanks system [26] and magnetic bearing system [27]. Advantages of the BS control are its capability to deal with the sudden changes in the characteristics of the system, the procedure to design the control law is systematic, and stability of the design despite modeling errors and external disturbances. In this section, the BS control is designed for the single-axis driven system. Let's define e_1 in term of the error that occurs between the position of actual angular and the position of the desired angular x_r :

$$\mathbf{e}_1 = \mathbf{x}_r - \mathbf{x}_1 \tag{8}$$

Taking the derivative of the error gives:

$$\dot{\mathbf{e}}_1 = \dot{\mathbf{x}}_r - \mathbf{x}_2 \tag{9}$$

In the context of BSC, let x_2 is chosen as the virtual control v. Then, the virtual control is substituted in Eq. (9) gives:

$$\dot{\mathbf{e}}_1 = \dot{\mathbf{x}}_r - \mathbf{v} \tag{10}$$

In this stage, the Lyapunov function is selected as:

$$V_1 = \frac{1}{2}e_1^2 \tag{11}$$

By differentiate V_1 results:

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1 (\dot{x}_r - v_1) \tag{12}$$

Taking v_1 as a virtual control yields:

$$\mathbf{v}_1 = \dot{\mathbf{x}}_r + \lambda_1 \mathbf{e}_1 \tag{13}$$

where $\lambda_1 > 0$,

Substituting v_1 in Eq. (12) gives:

$$\dot{V}_1 = -\lambda_1 e_1^2 \tag{14}$$

Define e_2 as the error between v_1 and x_2 :

$$e_2 = x_2 - v_1 (15)$$

Substituting v_1 as given in Eq. (13) in Eq. (15) obtains:

$$e_2 = x_2 - \dot{x}_r - \lambda_1 e_1 \tag{16}$$

Rearrange Eq. (16) gives:

$$x_2 = e_2 + \dot{x}_r + \lambda_1 e_1 \tag{17}$$

Substituting x_2 in Eq. (9) gives:

$$\dot{\mathbf{e}}_1 = -\mathbf{e}_2 - \lambda_1 \mathbf{e}_1 \tag{18}$$

Taking the time derivative of e_2 in Eq. (16) gives:

$$\dot{\mathbf{e}}_2 = \dot{\mathbf{x}}_2 - \ddot{\mathbf{x}}_r - \lambda_1 \dot{\mathbf{e}}_1 \tag{19}$$

Substituting \dot{x}_2 in Eq. (17) gives:

$$\dot{\mathbf{e}}_2 = \mathbf{f}(\mathbf{x}) + \mathbf{b}\mathbf{u} - \ddot{\mathbf{x}}_{\mathbf{r}} - \lambda_1 \dot{\mathbf{e}}_1 \tag{20}$$

To derive the control law, let Lyapunov function is selected as:

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \tag{21}$$

By taking the time derivative of V gives:

$$\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 \tag{22}$$

Substituting \dot{e}_1 and \dot{e}_2 in Eq. (22) gives:

$$\dot{V}_2 = e_1(-e_2 - \lambda_1 e_1) + e_2(f(x) + bu - \ddot{x}_r - \lambda_1 \dot{e}_1)$$
(23)

Simplified Eq. (23) gives

$$\dot{V}_2 = -\lambda_1 e_1^2 + e_2(f(x) + bu - \ddot{x}_r - \lambda_1 \dot{e}_1 - e_1)$$
(24)

Then, the control of the BS controller u_{bs} is selected as follows:

$$u_{bs} = \frac{1}{h}(-f(x) + \ddot{x}_r + \lambda_1 \dot{e}_1 + e_1 - \lambda_2 e_2)$$
 (25)

where $\lambda_2 > 0$

Applying the results of Eq. (25) in Eq. (24) gives;

$$\dot{V}_2 = -\lambda_1 e_1^2 - \lambda_2 e_2^2 \tag{26}$$

3.2. Synergetic Control

SG control is a control strategy useful for a large class of dynamical systems and, in particular, for nonlinear dynamical systems, which produces a stable control model based on the Lyapunov function. In particular, various applications of SG control can be found such as ball and beam systems [28] and magnetic levitation system [29]. In this section, the SG control is designed for the single-axis driven system. Let's define e_t as the tracking error between the position of the actual angular and the position of the desired angular x_r :

$$e_t = x_r - x_1 \tag{27}$$

Taking the first derivative of the error gives:

$$\dot{\mathbf{e}}_{t} = \dot{\mathbf{x}}_{r} - \mathbf{x}_{2} \tag{28}$$

Taking the second derivative of the error gives:

$$\ddot{\mathbf{e}}_{\mathsf{t}} = \ddot{\mathbf{x}}_{\mathsf{r}} - \dot{\mathbf{x}}_{\mathsf{2}} \tag{29}$$

Based on the tracking Error, the marco-variable φ is defined as follows:

$$\varphi = \dot{\mathbf{e}}_t + \mathbf{a}_1 \mathbf{e}_t \tag{30}$$

Differentiate φ results:

$$\dot{\varphi} = \ddot{\mathbf{e}}_{t} + \beta_{1}\dot{\mathbf{e}}_{t} \tag{31}$$

where $\beta_1(\beta_1 > 0)$ is a scalar tuning parameter

To ensure the stability of the system, the following equation is determined as:

$$\dot{\varphi} + \beta_2 \varphi = 0 \tag{32}$$

In Eq. (32), the tuning parameter $\beta_2(\beta_2 > 0)$ refers to the convergence rate towards the predefined manifolds. Substitute $\dot{\phi}$ from Eq. (31) in Eq. (32) obtains:

$$(\ddot{\mathbf{e}}_t + \beta_1 \dot{\mathbf{e}}_t) + \beta_2 \phi = 0 \tag{33}$$

Substitute Eq. (29) and Eq. (7) in Eq. (33) gives:

$$\ddot{x}_{1d} - f(x) - bu + \beta_2 \dot{e}_t + \beta_2 \phi = 0$$
 (34)

The control law of the SG controller (u_{sc}) is obtined by sloving Eq. (34):

$$u_{sc} = \frac{1}{h} (\ddot{x}_{1d} - f(x) + \beta_1 \dot{e}_t + \beta_2 \phi)$$
 (35)

3.3. High Gain Observer

Consider a class of single-input and single-output (SISO) second order nonlinear single-axis driven systems given by:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{36}$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + \mathbf{b}\mathbf{u} \tag{37}$$

where x_1 and x_2 are the states of the system, f(x) is a nonlinear functions, u is the control input, b > 0 is the input gain of the system. In some practical cases, only the output x_1 of the system is assumed to be measureable. However, for designing BS and SG controllers, all states of the system is required to design a control input u such that the output of the system x_1 follow a desired bonded reference input x_d . In such case, the high gain observer (HGO) is introduced for estimating the states. The estimation of the states based on HGO is given by [14]:

$$\dot{z}_1 = z_2 + h_1(x_1 - z_1) \tag{38}$$

$$\dot{z}_2 = f(x) + bu + h_2(x_1 - z_1) \tag{39}$$

where z_1 and z_2 are an estimation of the states x_1 and x_2 respectively. h_1 and h_2 are positive constant represents the design coefficients for the observer.

4. Circle Search Algorithm

Optimization algorithms are methods used to find the optimal solution among all of the potential solutions. In the context of engineering domain, optimization-algorithms are now become a useful sportive tool in solving numerous engineering problems [30]-[34]. Furthermore, determining the optimal values for controller design variables in order to produce control signals that enable the system to track the desired dynamic performance is a significant challenge. In this context, many researchers are turning to optimization algorithms instead of traditional trial-and-error methods to obtain the best adjustable controller parameters [35]-[39]. Inspired by the geometrical features of circles, Qais et al. [40] introduced a swarm optimization method named circle search algorithm (CSA). The circle is characterized by several geometric elements, including the diameter, center, and circumference, in addition to the lines tangent to it. Any set of points located in a certain geometric plane that form a closed curve so that they are equidistant from a central point (the center of the circle), is called a geometric circle. Such a closed curve is called the circumference of the circle. In Fig. 2, a circle whose center is (x_c) . The diameter of a Circle (D) can be defined as a straight segment connecting two distinct points located on the circumference of the circle, while the radius (R) is a straight segment connecting any point located on the circumference of the circle to its center. Suppose that a tangent line connecting the points (x_t, x_p) touches the circle at a point (x_t) , which represents the unique point of tangency between the line and the circumference of the circle, so it is well known that it is perpendicular to the radius (R) according to the geometric properties of the tangent. It can also be seen that the tangent (x_c, x_p) intersects the line segment (x_c, x_p) passing through the center. A right triangle will then be formed, and using trigonometric ratio functions, the tangent function of angle (θ) will be [40]:

$$\tan(\theta) = \frac{R}{x_p - x_t} \tag{40}$$

While $R = x_t - x_c$, this gives:

$$\tan(\theta) = \frac{x_t - x_c}{x_p - x_t} \tag{41}$$

$$x_t - x_c = \tan(\theta) \times (x_p - x_t)$$
 (42)

$$x_t - x_c = \tan(\theta) \times (x_p - x_t)$$
(43)

$$x_t = x_c + \tan(\theta) \times (x_p - x_t) \tag{44}$$

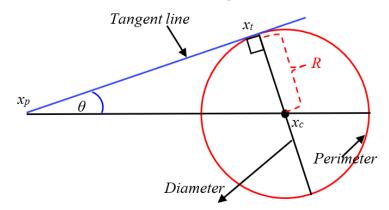


Fig. 2. Terminologies of the geometric circle

From the general mathematical principles of circles given in the above, it is clear that as the radius of the circle decreases, the tangent angle will decreased so that the tangent gradually will

approach to the center of the circle, as shown in Fig. 3 a. In the CSA algorithm, it is assumed that the center of the circle is the target point for the optimal solution and any random point is the tangential point (x_t) . The random point's approach to the solution is achieved by decreasing the tangent line angle (θ) , as illustrated in Fig. 3 b, the tangential point (x_t) serves as the search agent for the CSA, while (x_c) is conceptualized as the optimal solution of the algorithm.

It is obvious from Fig. 3 that; the CSA search agent continuously adjusts the tangential point's position in the direction of the center point. However, to prevent the CSA from becoming trapped within the boundaries of a local solution, the contact point is randomly altered by varying the angle in a stochastic manner. The steps of the CSA can be described as follows [41]:

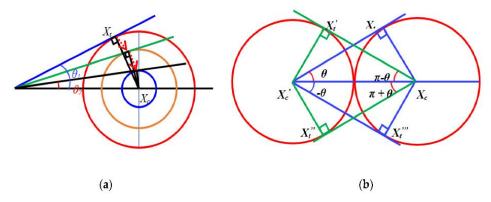


Fig. 3. The sequence of the CSA: (a) exploitation and (b) exploration

Step 1: Initialization: It is an essential step that must be implemented in CSA to ensure equal randomness of all dimensions of the search agent. In the previously published code, it may be possible to obtain very surprising and unexpectedly fast results from algorithms due to the fact that most of them randomly distribute dimensions unevenly. Subsequently, the search agents are initialized within the bounds of the search space, specifically between the upper limit values (UV) and lower limit values (LV), as defined in Eq. (45):

$$x_t = LV + r \times (UV - LV) \tag{45}$$

where r is any real random number between 0 and 1.

Step 2: Reposition the search agent to a new location: The search agent (x_t) takes a new location according to the best evaluated location (x_c) , as shown in Eq. (46):

$$x_t = x_c + \tan(\theta) \times (x_c - x_t) \tag{46}$$

The angle (θ) has a significant impact on exploring and employing the CSA and can be calculated as follows:

$$\theta = \begin{cases} w \times Rand & \text{Iter} > (c \times Maxiter) \text{ (escape from local stagnation)} \\ w \times p & \text{otherwise} \end{cases}$$
 (47)

$$w = w \times Ran - w \tag{48}$$

$$a = \pi - \pi \times \left(\frac{Iter}{Maxiter}\right)^2 \tag{49}$$

$$p = 1 - 0.9 \times \left(\frac{Iter}{Maxiter}\right)^{0.5}$$
 (50)

where Rand is an arbitrary value within the range of (0 to 1), Iter is the iteration counter, Maxiter is the maximum iterations number, and c is a constant within the range of (0 to 1), which proportion

of the maximum of iterations. According to Eq. (49), the variable a can be chosen in the range from $(\pi \text{ to } 0)$, whereas the variable p, as indicated in Eq. (50), can vary from (1 to 0). Consequently, the changing in the angle q is from $(-\pi \text{ to } 0)$.

Two different cases for accomplished the CSA which are: In the case of Iter > (c.Maxiter): the angle θ is determined by ($\theta = w \times Rand$) for all the time, which is used to enhance the exploration search of the CSA. On the other hand, if Iter < (c.Maxiter), in this case, the angle θ is determined by ($\theta = w \times p$) for all time, which is employed to enhance the process of exploitation search.

5. Simulation Results

In this section, the effectiveness of proposed HGO in estimating the actual states of the system and designing a BS and SG controllers is evaluated. For this purpose, the numerical simulation based on MATLAB program has been conducted. MATLAB is an efficient tool that can be used to design and simulate different control algorithms. Thanks to its sophisticated numerical solvers which can handle complex nonlinear differential equations. The equations of motion that are given in Eq. (5) and Eq. (6) of the system have been coded to capture the open loop dynamic of the system. The numerical values of the parameters of the considered system are given in Table 1 [11].

Table 1. Numerical values of system parameters

Parameter	Value
Acceleration due to the gravity (g)	9.81
Moment of the inertia (J)	0.0106
Mass of the propeller (m)	0.36
Viscous damping coefficient (C)	0.0076
Motor coefficient (K _m)	0.0296
Suspension-to-mass center distance (d)	0.03

CSA algorithm has been employed to adjust the value of the deign parameters of the HGO (h_1 and h_2), the BS controller (λ_1 and λ_2) and the SG controller (a_1 and a_2) and reported in Table 2. The Integral Absolute Error (IAE) criterion was selected as a cost function for the CSA to in the optimization process as given by [42]-[44]:

$$IAE = \int_0^{t_s} |e| dt$$
 (51)

where t_s is the time of the simulation and e is the output the error between the angular position and the desired angular position.

Table 2. Optimal value of configuration parameters based on CSA

Controller	Parameters	Values
HGO	h ₁	34.6
	h_2	20.24
BS controller	λ_1	45.26
	λ_2	42.25
SG controller	β_1	38.6
	β_2	25.4

In the first scenario, the open-loop test of the system has been simulated for estimating the system's states. The initial value of the states of the system $(x_1 \text{ and } x_2)$ was set to (0.1 and -0.1) respectively. It can be seen form Fig. 4 and Fig. 5 that the HGO is successfully estimating all states of the system. In the second scenario, the controlled system based on BS and SG controllers is subjected to a reference angle position (x_r) which is generated based on the following the equation.

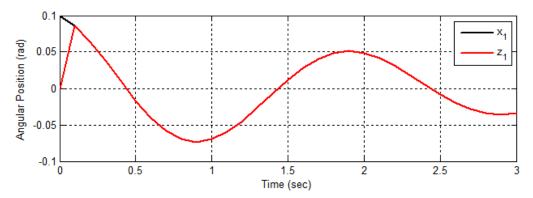


Fig. 4. The performance of HGO to estimate x_1

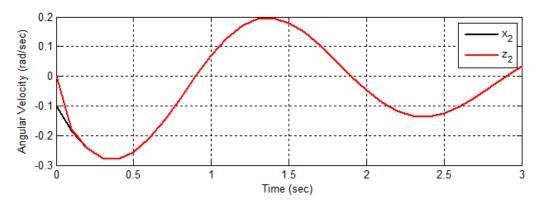


Fig. 5. The performance of HGO to estimate x_2

$$x_r = \sin(2t) + \cos(3t) \tag{52}$$

The tracking performance to the desired trajectory of the angle x_1 based on the BS and SG controllers is shown Fig. 6. It is important to mentation that the control law of the BS and SG controllers was designed based on the estimation states instead of the actual states. It can be seen based on Fig. 6 that the BS controller exhibits better performance than SG controller regarding to the fast convergence to the desire reference. As shown in Table 3, the evaluation uses the IAE to compare two nonlinear controllers. The BS controller outperforms the other by producing smaller tracking errors in angular position, resulting in a lower IAE.

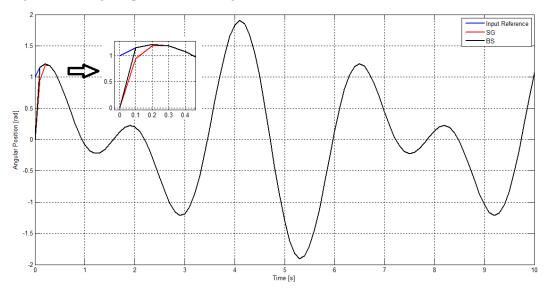


Fig. 6. Tracking performance for system controlled by BS and SG

Table 3. Performance evaluation of BS and SG controllers based HGO

Controller	IAE
BS controller	1.003
SG controller	1.245

6. Conclusion

The single-axis driven system is a variation of the traditional simple pendulum, enhanced by the addition of a propeller motor at its end. The torque produced by the motor drives the pendulum's up-and-down motion. To control the single-axis driven system, two model-based controllers including a backstepping (BS) and synergetic (SG) controllers are proposed. Unlike the previous work, the control signal of the two controllers is designed based on the estimated state that is obtained by the high gain observer (HGO). To ensure a suitable basis of comparison, the design parameters of the two control algorithms are optimized by a circle search algorithm (CSA) using Integral Absolute Error (IAE) criterion. The numerical computer simulation based on MATLAB program results revealed that he HGO is successfully estimating all states of the system. Besides, in terms of tracking control, the BS controller shows better performance by achieving a good speed convergence than SG controller. Motivated by these results, the potential future research directions could be using the BS controller based on HGO to control systems where there is a difficulty to access to the actual states of the system. Further enhancement can be done to the BS controller by augmented it with other control algorithms.

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