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A Systematic Review of Inverse Kinematics Methods for Fixed-Base Serial Manipulators: Analytical, Numerical, and Machine Learning Methods

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ABSTRACT

Inverse kinematics is essential for precision tasks in fixed-base serial robots, such as surgical robotics or high-speed manufacturing, where delays or errors can have critical consequences. Current inverse kinematic methods face a fundamental trade-off: analytical solutions are fast but limited to spherical-wrist manipulators, while numerical and AI-based approaches sacrifice speed for generality. Despite prior reviews comparing performance metrics, no study provides a unified quantitative framework to guide method selection based on robot structure or application requirements. This systematic review addresses this lack of (1) quantitatively contrasting (response time, accuracy) analytical, numerical, and AI-based methods using studies in fields such as industrial robotics, medicine, and collaborative spaces and (2) identifying optimal hybrid strategies for real-time applications such as path planning. Using PRISMA, we analyzed 47 peer-reviewed articles from Scopus/Web of Science between 2019-2024, excluding algorithms for continuous, parallel, or mobile robots to focus solely on fixed-base serial architectures; selecting topics like 'inverse kinematics and serial robots and analytical or numeric or machine learning methods'. The review reveals that 32% of the analyzed methods are numerical, while 30% are AI-based approaches, reflecting the growing interest in data-driven solutions for IK problems; this scenario highlights the implementation of these methods given the limitations of analytical methods. Moreover, 56% of the non-analytical approaches achieve an accuracy better than 0.01 mm; and about 70% of these approaches have response times exceeding 20 ms or don´t evaluate the metric, highlighting a critical bottleneck for real-time use. We conclude that hybrid IK methods, combined with standardized validation protocols, are essential for critical applications like robotic surgery. Future work must address benchmarking gaps, especially in AI-based IK, to enable reliable adoption in industry.

1. Introduction

Industry 4.0 has driven significant advancement in manufacturing and industrial automation, thanks to digital and physical technologies such as the Internet of Things (IoT) and intelligent automation systems [1]. In this context, robotics has established itself as an essential component, optimizing processes, improving efficiency and expanding the productive capacities of industries [2]. The adoption of robots in industrial environments has proven to be highly beneficial, generating a notable increase in productivity and, in the medium term, in company profits. Moreover, this technology has transcended the manufacturing field, meeting new challenges such as the positioning of the end effector in sectors such as robotic surgery and rehabilitation in hospitals [3]. A vitally important challenge in achieving this is solving the inverse kinematics (IK) problem, as its application is key to ensuring that the desired Cartesian motions are manifested in the end effector. Methods for solving the IK allow the calculation of the joint positions and orientations required to place the end effector in a desired position and orientation [4], [5]. Accuracy in these calculations is crucial for structural design, motion planning and dynamic analysis of manipulators [6]. When evaluating these methods, is common to consider three variables: efficiency, effectiveness and variety of solutions. Efficiency refers to the computation time required to obtain the solution. Effectiveness implies that the solution found is accurate within predefined ranges. Finally, the variety of solutions ensures that the method can identify multiple joint trajectories without incurring singular configurations, providing greater flexibility and adaptability in different applications [7].

Analytical methods have been the predominant choice for solving IK in manipulators with up to six degrees of freedom (DOF) due to their ability to provide accurate and efficient solutions when the robotic system allows an algebraic formulation. This is because, in such cases, analytical methods can directly derive joint positions without resorting to numerical approximations or iterative solutions [5]. However, its application is usually limited to robots up to six DOF, even sometimes this method does not allow to find the solution for 5 DOF robots [8], [9] because the analytical solutions can be difficult to find or even non-existent when the robot configurations do not adapt to the conditions necessary to apply Pieper's geometrical rules (the axes of the last three joints cross at a point), i.e. they do not have a spherical wrist [10], [11]. To overcome this limitation, numerical methods, such as optimization algorithms or Newton-Raphson techniques [12], have proven useful for solving IK in more complex robots. However, they present a higher computational cost and may converge to non-optimal local solutions. In addition, these methods are often applied to robots with geometries common to a single robot family, limiting their generalization [13]. While traditional approaches, both analytical and numerical, have proven to be effective in certain contexts, the complexity of some robotic configurations, such as redundancies and non-spherical wrists, require methods that integrate the geometric and dynamic properties of the system. In this scenario, the analytical method based on screw theory is positioned as a robust alternative. This approach [14], based on the use of "screws" to represent the movements, makes it possible to derive accurate solutions for the position and orientation of the joints, especially in robots with spherical wrist. With the rise of artificial intelligence (IA) or machine learning methods (LM) have gained prominence. These approaches avoid analytical analysis, relying on large volumes of data for training [15]. Despite their advantages, they have the disadvantage of requiring a new training each time the physical parameters of the robot change [16], moreover, these models usually provide only one solution, which is not always the optimal one. Another type of algorithms are heuristic algorithms, which are based on population rules. Like numerical approaches, these algorithms converge to a solution that is at a distance from the objective. These algorithms differ from IA algorithms; while heuristics focus on search and optimization [17], IA algorithms generate a probabilistic model, although this sometimes lacks explainability.

With all these advances, existing reviews on IK have critical shortcomings in that, they separately summarize analytical and numerical [18], and learning-based methods [19] and often do not address systematic criteria for selecting algorithms based on robotic wrist structures (e.g., spherical vs. non-spherical). This lack creates a critical gap: without a selection framework based on quantitative comparisons, researchers face uncertainty in choosing among methods, risking suboptimal or inapplicable solutions for specific robots. For example, analytical methods remain limited to simple structures (e.g., non-redundant, non-cylindrical robots), while numerical and machine learning approaches suffer from generalization or convergence problems. In addition, the lack of standardized benchmarks to compare the performance of different IK methods compounds the problem, as noted in recent studies. Although previous reviews acknowledge computational inefficiency or structural limitations, none critically assess why these gaps persist or propose ways to address them (e.g., integrating analytical accuracy with numerical flexibility). This review directly addresses these needs by synthesizing method-specific limitations into a unified decision framework for algorithm selection and identifies understudied hybrid approaches that could bridge the trade-off between response time, accuracy, and solution variety.

With the above limitations, in this article, a systematic review of the literature using the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) method is performed [20]. Following the PRISMA methodology—a gold standard for systematic reviews—this study ensures transparency and reproducibility through structured screening, selection, and synthesis of literature, minimizing bias in evidence collection. This review systematically analyzes 47 studies published between 2019 and 2024, focusing on IK resolution methods for fixed-base serial robots with spherical and non-spherical wrists. The findings provide actionable insights for industries adopting robotic automation, particularly in selecting IK methods that balance precision, computational speed, and adaptability to diverse wrist structures—critical for applications like high-precision manufacturing and medical robotics. Following the PRISMA methodology, it seeks to answer the following key questions: (1)What are the most commonly employed analytical, numerical, machine learning or IA methods for solving IK in robots? (2)What are their advantages and limitations depending on the robotic structure and accuracy requirements?

This review makes three primary contributions: (1) a wrist-structure-aware framework for IK method selection, (2) a synthesis of hybrid approaches to balance accuracy-flexibility trade-offs, and (3) standardized metrics for benchmarking IK algorithms—addressing gaps in generalization, comparability, and industrial applicability identified in prior research.

The paper is structured as follows: Section 2 details the methodology used in this study. Section 3 (Results and Discussion) presents and analyzes key findings, synthesizing them into a decision-making framework for selecting the most suitable IK algorithm based on critical performance variables such as precision, response time, and computational efficiency. Finally, Section 4 (Conclusions) summarizes key insights, discusses practical implications, and proposes future research directions to optimize algorithm selection for structural analysis.

2. Method

2.1. Eligibility Criteria

The PRISMA 2021 method is used to perform a systematic literature review of methods for finding the IK, such as: analytical, numerical, optimization and/or IA-based, when applied to serial robots. The specific methodology followed is described in the following sections.

The period 2019-2024 was selected to capture recent advances in computational methods and to avoid technological obsolescence. Studies were eligible if they applied to numerical algorithms, such as iterative methods, in addition to analytical or geometric algorithms, as well as IA-based and heuristic approaches. Other eligibility criteria included the relevance of the algorithm for solving inverse kinematics in serial robots in terms of accuracy, response time, or variety of solutions. Although planar, parallel, mobile or continuous robots have important applications in medicine and bioengineering (minimally invasive surgery or rehabilitation), their exclusion is due to key structural differences, minimally invasive surgery or rehabilitation), their exclusion is due to key structural differences: (1) Fixed-based serial robots feature open kinematic chains and independent degrees of freedom, which simplifies mathematical modeling and allows algorithms to be compared under homogeneous criteria; (2) Inverse kinematics solutions for non-serial robots (such as parallel) involve additional mechanical constraints (closed loops) that require different computational approaches. This delineation ensures consistency in the evaluation of methods for the specific problem of serial robots, which represent a standard in industrial and surgical applications (universal robots family).

2.2. Information Sources

The Scopus and Web of Science reference database were electronically searched for eligible studies. The search was conducted from January 2019 to December 2024.

2.3. Search Strategy

The search string used in Scopus and Web of Science was designed to identify relevant studies on inverse kinematics in serial robots. The main concepts selected were “inverse kinematic”, “robot” and terms related to methods, algorithms or techniques used in the resolution of inverse kinematics. In addition, the categories of interest “numeric”, “neural net” “analytical”, “iterative”, “geometric” were included as they represent key approaches in this research area. Additional limits were applied according to the eligibility criteria, selecting studies published in the areas of engineering, computer science and mathematics. In addition, only scientific articles (ar), published in journals, were considered. This strategy ensures that the selected studies fit the areas of interest and meet the established methodological and publication criteria. The search strings used are listed below.

2.3.1. Scopus

TITLE-ABS-KEY (((”inverse kinematic”) AND (” robot ”) AND (”Methods” OR ”algoritms” OR ”tecniques” OR ”approach”) AND (”Numeric” OR ”neural net” OR ”machine learning” OR ”IA” OR ”Analytical” OR ”geométric” OR ”iterative”))) AND (LIMIT-TO (SRCTYPE,”j”)) AND (LIMIT-TO (DOCTYPE,”ar”)) AND (LIMIT-TO (PUBYEAR,2019) OR LIMIT-TO (PUBYEAR,2020) OR LIMIT-TO (PUBYEAR,2021) OR LIMIT-TO (PUBYEAR,2022) OR LIMIT-TO (PUBYEAR,2023) OR LIMIT-TO (PUBYEAR,2024)) AND (LIMIT-TO (LANGUAGE,”English”))

2.3.2. Web of Science

TS=(”inverse kinematic” AND robot AND (numeric OR ”neural network” OR ”machine learning” OR ia OR analytical OR iterative OR geometric))
AND SU=(Engineering OR ”Computer Science” OR Mathematics)
AND DT=(Article)
AND PY=(2019 OR 2020 OR 2021 OR 2022 OR 2023 OR 2024)

2.4. Selection Process

The articles obtained from the databases and the manual search were imported into the Mendeley web library, used as the main tool for the management and organization of references. Each article was reviewed to verify its compliance with the eligibility criteria, and after an initial review, the final set of studies comprising this review was selected. In total, 47 articles were included, the selection process of which is presented in the flow chart in Fig. 1.

A sample of 99 records in Scopus and 77 in WOS were identified from the bibliographic search. A total of 9 articles were eliminated because they were duplicates; reading the abstract did not contribute to the objective of the review (methods to find the ik in serial robots in fixed base with their respective comparison or if the algorithm is new). Then 15 of the articles that talked about these methods were about mobile, continuous, planar, parallel robots, which, as explained above, is not the focus of the review. The workflow with the detailed process is shown in Fig. 1.

A cloud-based collaborative spreadsheet (Google Sheet) was used to record the data from the selected studies. The document was organized in a state-of-the-art matrix, where each row corresponded to a study and the columns indicated the information to be analyzed. The studies were to be reviewed in full and the relevant columns of the matrix were to be completed.

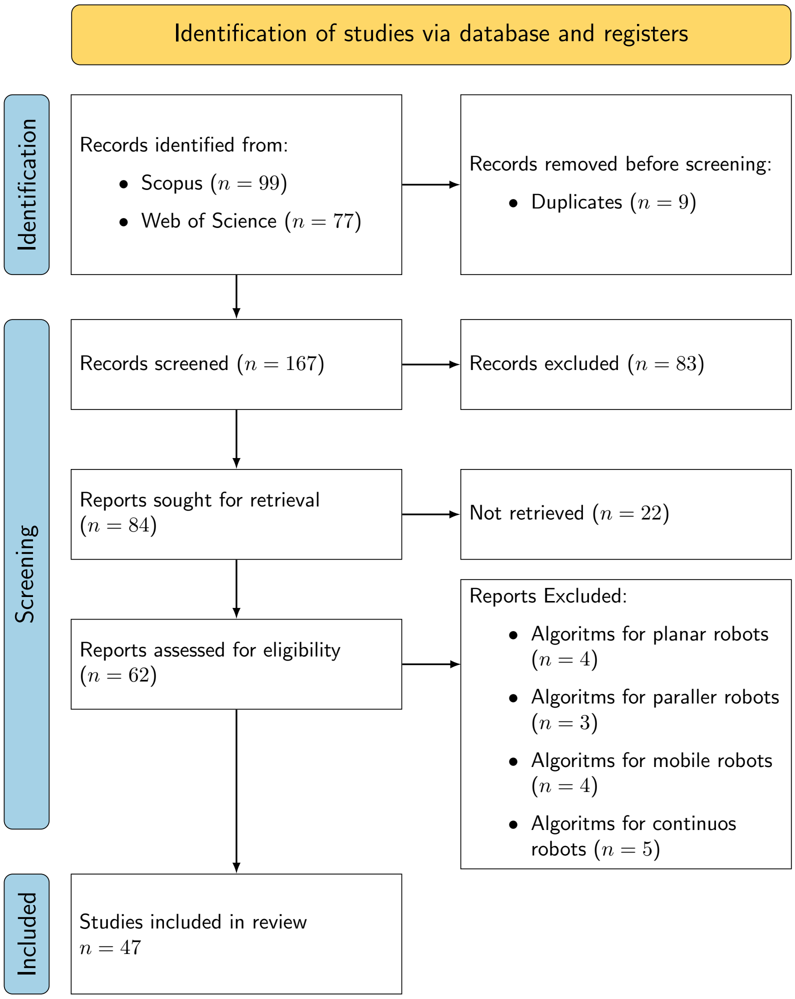

Fig. 1. Selection process performed in this review. This flow chart shows that from an initial value of 157 articles, a total of 47 articles were selected for review.

2.5. Information Fields

The columns defined in the collaborative spreadsheet corresponded to the key variables for which data were sought. Specific columns included: type of method (analytical, numerical, neural network-based), robot degrees of freedom (number of joints), robot structure (spherical wrist, non-spherical wrist, continuous robots), algorithms compared to (analytical algorithms, numerical methods, heuristic and machine learning), variables compared (accuracy, computational time, number of solutions), types of trajectories (if applicable, linear or curved trajectories), types of applications (industrial, medical, educational), performance results, validation methodologies, and challenges and future work.

2.6. Limitations

This review acknowledges several limitations, one of which is the exclusion of IEEE Xplore as a data source may have underrepresented industrial applications, given this database's large coverage of engineering and robotics literature. Restricting the studies to English-language publications could omit relevant contributions published in other languages. In addition, the selection of titles and abstracts might have overlooked methodological details or innovations described only in the full texts.

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Although manual searches of conference proceedings (ICRA/IROS) helped to mitigate these gaps, they could not completely eliminate possible biases or omissions in the study selection process.

3. Results and Discussion

This systematic review identifies and analyzes three main methodological approaches for computing IK in serial robots: analytical, numerical, and IA methods, with an additional emerging category of hybrid techniques that integrate multiple approaches. This methodological classification, derived from the analysis of 47 studies published between 2019-2024, reveals distinctive patterns in applicability, accuracy, and computational efficiency across different robotic configurations. As shown in Table 1, there are significant variations in how these methods perform across key performance metrics (accuracy, response time, and solution multiplicity) when applied to robots with different DOF and wrist configurations. This comparative framework allows for a critical assessment of methodological strengths, limitations, and application contexts that previous reviews by [18] and [19] have not comprehensively addressed.

Table 1. Comparison of inverse kinematics methods for robots

Author	Type method	DOF	Type of wrist	Comparative algorithms	Accuracy	Response time	Number of Solutions
[21]	Analytical	6	Non-spherical wrist	Numerical	Exact	Not applicable	8-16 solutions
[22]	Analytical-Numerical	> 6	Spherical wrist	New Algorithms	Exact	Not applicable	Unique
[23]	Numerical	6	Non-spherical wrist	New Algorithms	Exact	Not applicable	8-16 solutions
[24]	AI	< 6	Non-spherical wrist	Traditional-Analytical	0.1-1 mm	Not applicable	8-16 solutions
[25]	Numerical	6	Spherical wrist	Numerical	0.1-1 mm	Not applicable	Unique
[26]	Numerical	> 6	Spherical wrist	Numerical	0.01-0.1 mm	0.001-0.005 s	Unique
[27]	Hybrid-Analytical-AI	6	Spherical wrist	Traditional-Analytical	0.01-0.1 mm	Not applicable	Unique
[28]	Analytical	6	Spherical wrist	New Algorithms	Exact	Not applicable	Unique
[29]	Hybrid-Analytical-AI	> 6	Spherical wrist	New Algorithms	Exact	0.02 - 0.5 s	Unique
[30]	Analytical-Numerical	6	Non-spherical wrist	Numerical	Exact	0.005-0.02 s	8-16 solutions
[31]	Analytical	6	Spherical wrist	New Algorithms	Exact	Not applicable	Unique
[32]	Analytical	< 6	Non-spherical wrist	Traditional-Analytical	Exact	Not applicable	Unique

Continued on next page

Author	Type method	DOF	Type of wrist	Comparative algorithms	Accuracy	Response time	Number of Solutions
[33]	Numerical	6	Non-spherical wrist	Numerical	0.1-1 mm	0.02-0.05 s	Unique
[34]	AI	> 6	Spherical wrist	Numerical	0.1-1 mm	0.02-0.05 s	Unique
[35]	Heuristic	> 6	Spherical wrist	Heuristic	Exact	0.02-0.05 s	Unique
[36]	Heuristic	> 6	Spherical wrist	Heuristic	0.01-0.1 mm	0.02-0.05 s	Unique
[37]	Analytical	< 6	Spherical wrist	Traditional-Analytical	Exact	Not applicable	Unique
[38]	Numerical	> 6	Spherical wrist	Numerical	0.1-1 mm	0.001-0.005 s	Unique
[39]	Hybrid-Analytical-AI	6	Non-spherical wrist	Traditional-Analytical	Exact	Not applicable	Unique
[40]	Analytical	> 6	Spherical wrist	Traditional-Analytical	Exact	Not applicable	Unique
[41]	Numerical	6	Non-spherical wrist	Numerical	0.01-0.1 mm	Not applicable	Unique
[42]	Numerical	6	Non-spherical wrist	Numerical	0.01-0.1 mm	0.005 - 0.02 s	Unique
[43]	Numerical	6	Non-spherical wrist	Numerical	Exact	0.005 - 0.02 s	Unique
[44]	Numerical	6	Spherical-Non-spherical	Traditional-Analytical	0.01-0.1 mm	0.005 - 0.02 s	Unique
[45]	Heuristic	< 6	Non-spherical wrist	Heuristic	Exact	0.02-0.05 s	Unique
[46]	Numerical	> 6	Spherical wrist	Numerical	Exact	0.005-0.02 s	Unique
[47]	Numerical	> 6	Spherical-Non-spherical	Numerical	0.01-0.1 mm	0.02-0.05 s	8-16 solutions
[48]	Analytical	> 6	Spherical wrist	Traditional-Analytical	Exact	Not applicable	8-16 solutions
[49]	Heuristic	6	Non-spherical wrist	Heuristic	0.01-0.1 mm	Not applicable	Unique
[50]	Analytical-Numerical	> 6	Spherical wrist	Traditional-Analytical	0.01-0.1 mm	Not applicable	Unique
[51]	Hybrid-Analytical-AI	6	Non-spherical wrist	Numerical	0.01-0.1 mm	Not applicable	Unique
[52]	AI	6	Spherical wrist	AI	0.01-0.1 mm	Not applicable	Unique
[53]	Analytical	6	Spherical wrist	Traditional-Analytical	Exact	0.005-0.02 s	Unique

Continued on next page

Author	Type method	DOF	Type of wrist	Comparative algorithms	Accuracy	Response time	Number of Solutions
[54]	AI	< 6	Spherical wrist	AI	0.1-1 mm	Not applicable	Unique
[55]	AI	< 6	Spherical wrist	AI	0.01-0.1 mm	Not applicable	Unique
[56]	Analytical	6	Non-spherical wrist	Analytical	Exact	0.02-0.05 s	Unique
[57]	Hybrid-Analytical-Numerical	6	Non-spherical wrist	Numerical	0.001-0.01 mm	0.02-0.05 s	Unique
[58]	Numerical	6	Non-spherical wrist	Numerical	0.01-0.1 mm	0-0.005 s	Unique
[59]	Numerical	6	Non-spherical wrist	Numerical	Exact	Not applicable	Unique
[60]	AI	> 6	Spherical wrist	New Algorithms	0.01-0.1 mm	0.02-0.05 s	Unique
[61]	AI	< 6	Non-Spherical wrist	Traditional-Analytical	0.01-0.1 mm	-	Unique
[62]	AI	< 6	Spherical wrist	AI	0.001-0.01 mm	0.02-0.05 s	Unique
[63]	Analytical	> 6	Spherical wrist	New Algorithms	Exact	-	8-16 solutions
[64]	Numerical	6	Non-Spherical wrist	Numerical	Exact	0.02-0.05 s	Unique
[65]	AI	6	Spherical wrist	Numerical	0.1-1 mm	-	Unique
[66]	AI	< 6	Non-Spherical wrist	Analytical	0.1-1 mm	-	Unique
[67]	AI	6	Spherical wrist	AI	0.01-0.1 mm	-	Unique

3.1. Types of Methods

Analytical methods [21], [28], [31], [32], [37], [40], [48], [53], [56], [68] provide closed-form solutions, particularly for robots with spherical wrists. While [21] identifies gaps in complete IK solutions, their geometric approach enhances trajectory accuracy when integrated with global models like [28]. Quaternion-based methods, such as [31], simplify rotations and could optimize [68]'s sequential quadratic programming for niche applications (e.g., nuclear cleanup). Extensions to confined-space robots, like Jiang's 1P4R design [32], may further benefit from quaternion generalizations. Hybrid analytical-optimization approaches are exemplified by [37], [48], [56]: Singh combines SLSQP with algebraic solutions, Dou validates ROS-integrated vision applications, and Wang's least-squares approximation transforms non-spherical wrists into spherical equivalents for exact solutions. Singularity avoidance in hyper-redundant manipulators [40] and screw-theory-based 6R robots [53] underscores the trade-off between precision and computational cost in analytical IK.

Numerical and iterative methods [23], [25], [26], [33], [38], [41]–[44], [46], [47], [58], [59], [69], [70] prioritize computational efficiency and constraint adaptability. Zhang's uniqueness domain (UD) segmentation [23] improves high-dimensional trajectory tracking, complementing Yu's real-time bone-drilling compensation [25]. Niu's FABRIK adaptation [26] for hyper-redundant robots highlights the need for confined-space solutions—a theme echoed in Lee and Colan's RCM-constrained surgical robotics [33], [38]. Zhou and Chen [42],

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[43] eliminate initial guess dependencies via polynomial methods, while Xie and Marić [44], [46] enhance numerical stability with convex optimization. Giamou [47] integrates collision avoidance into this framework, achieving fast convergence.

Machine learning (ML) and population-based algorithms [24], [34]–[36], [45], [49], [52], [54], [55] address nonlinearities but face real-time adaptability gaps. Vu [34], [54], [55] combines neural networks with redundancy for dynamic tasks, while Bai's hybrid FABRIK-ANN method [24] reduces positional errors. Population-based approaches (e.g., Slim's Bat Algorithm [35], Danaci and Zhao's PSO variants [36], [45]) mitigate singularities but struggle with complex configurations. Yiyang's inertia-adjusted PSO [49] improves robustness, though tuning remains challenging. Jumma's neural networks [52] excel in repetitive tasks but lack dynamic adaptability.

Hybrid methods [22], [27], [29], [30], [39], [50], [51], [57] merge analytical and numerical strengths. Wang and Pan [22], [50] optimize redundant DOFs for energy efficiency, while Chen's mathematical simplifications [30], [57] enhance joint complexity handling. ML-augmented hybrids (e.g., Ojer [27], Nguyen [39]) face resource constraints, whereas Huang's geometry-neural fusion [29] pioneers real-time multi-DOF trajectory optimization.

3.2. Variables Analyzed

The articles reviewed in this study analyze IK performance through three primary variables: accuracy, response time, and solution diversity, as illustrated in Fig. 2. The graph reveals that methods with exact solutions often have response times greater than 0.005 seconds or, in many cases, do not even report this metric. In addition, approximately 50% of the records without response time information show accuracies greater than 0.01 mm. The visualization also shows that numerical and artificial intelligence-based algorithms tend to be located in this zone, characterized by accuracies greater than 0.01 mm and response times greater than 20 milliseconds, suggesting a relationship between the computational nature of these methods and the time cost associated with their computations.

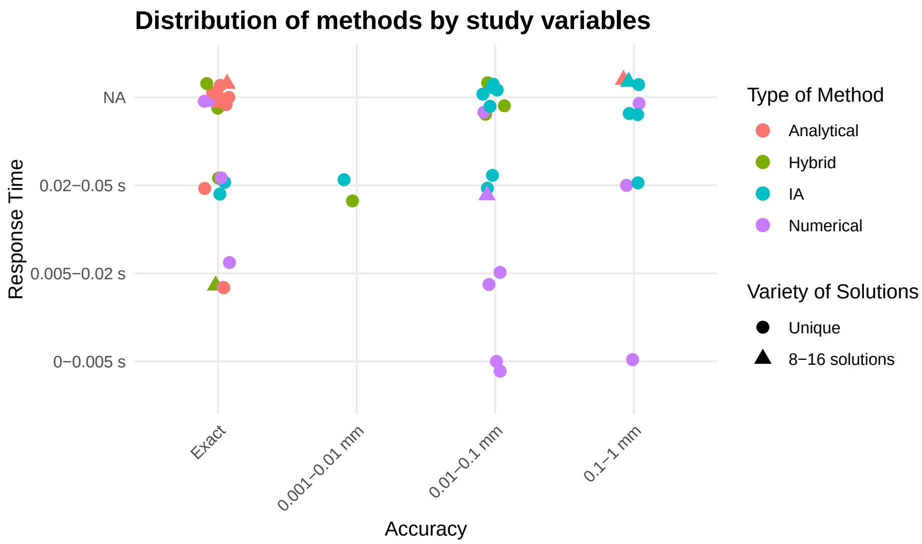

Fig. 2. Relationship between the type of algorithm, accuracy, response time and variety of solutions. AI-based algorithms have response times greater than 0.02 s.

Methodological Trade-offs. Analytical methods, such as those in [28], [31], prioritize exact precision and trajectory optimization, yet differ in their focus: the former emphasizes motion smoothness, while the latter reduces computational complexity. In contrast, numerical approaches [33], [38], [42], [55], [59] prioritize generalizability and speed, though they struggle with accuracy under dynamic conditions (e.g., shifted wrist configurations). Hybrid methods, like those of [51], [52], blend neural networks with analytical models to balance real-time adaptability and precision, reflecting a growing trend toward integrated solutions.

Performance Benchmarks. Recent studies highlight stark contrasts in performance metrics. For instance, [26] achieves sub-millimeter accuracy (0.01–1 mm) with response times under 0.005 s using numerical methods, ideal for hyper-redundant robots. Conversely, [34] reports lower accuracy (5–20 mm) and slower responses (0.02–0.5 s) with neural networks, favoring flexibility over precision. Energy efficiency emerges as a secondary criterion in [29], which optimizes trajectories via pseudo-attractor theories, while [44] leverages distance geometry for high-accuracy convergence in constrained workspaces.

Solution Diversity. The trade-off between precision and solution variety is exemplified by [21] (geometric validation of 8–16 configurations) and [23] (configuration-space decomposition for computational efficiency). Task-specific demands further polarize approaches: [25] favors single-solution precision for medical drilling, whereas [39] employs statistical inference to enhance adaptability in industrial settings. This spectrum underscores that optimal IK strategies are context-dependent, balancing accuracy, speed, and flexibility to meet application requirements.

3.3. Type of Wrist

The configuration of the robot's wrist—whether spherical or non-spherical—fundamentally influences the selection and effectiveness of IK methods. Our analysis reveals distinct methodological patterns based on wrist type, with significant implications for precision, computational requirements, and practical applications.

The Fig. 3 shows a remarkable influence of both degrees of freedom (DOF) and robot structure (spherical vs. non-spherical wrist) on accuracy and response time. First, robots with exactly 6 DOF exhibit greater variety in accuracy and response time metrics. Many of them achieve exact accuracy, although they also concentrate in the 0.01-0.1 mm and 0.1-1 mm ranges.

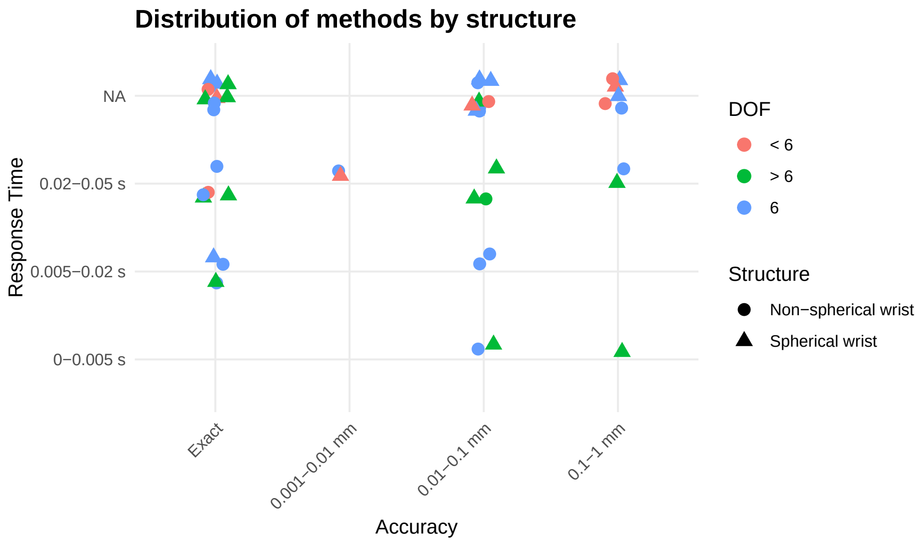

Fig. 3. Relationship between DOF, robot structure and performance of inverse kinematics algorithms.The predominance of analytical methods for spherical wrist configurations contrasts with the pattern observed for nonspherical wrists, reflecting the mathematical decomposability advantage offered by these configurations. This distribution reveals a strong preference for numerical approaches in dealing with the mathematical complexities of nonspherical wrist kinematics

However, for the most part, these values are not accompanied by response time information, which limits the evaluation of their temporal performance. On the other hand, robots with more than 6 DOF tend to show high accuracy (even exact or less than 0.1 mm) along with response times generally above 0.02 seconds, especially in spherical wrist structures. This may be due to the additional computational complexity involved in handling more degrees of freedom, which slows down the computation, despite allowing higher accuracy.

In terms of structure, it is seen that robots with a spherical wrist dominate the high-precision regions, even when response times are higher, reinforcing the idea that these configurations allow for greater spatial dexterity at the cost of greater computational effort. In contrast, non-spherical wrist robots, while also exhibiting accurate precision in several cases, tend to be more associated with short response times (e.g., in the 0-0.005 s range), possibly due to lower computational load or geometric simplicity. From the above, it is argued that higher DOF such as the use of spherical wrists appear to favor accuracy in solving inverse kinematics, albeit at the cost of

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longer response times. This suggests a trade-off between structural complexity and time efficiency that should be carefully considered in the design and selection of algorithms for robots of different architectures.

Spherical Wrist Robots

The papers investigate IK algorithms applied to spherical wrist robots, where three concurrent joint axes intersect at a single point. This geometric property enables mathematical decomposition of the positioning and orientation problems, a significant advantage compared to non-spherical configurations. As shown in Fig. 3, the scientific community has maintained consistent interest in spherical wrist robots over the past five years, with analytical methods representing 35% of approaches, numerical methods 25%, heuristic-AI methods 20%, and hybrid approaches 20%.

When examining high-DOF spherical wrist robots (¿6 DOF), three distinct methodological clusters emerge. [22], [26], and [29] focus on redundancy management, proposing innovative solutions that address operational complexity. [22] integrates dynamic load indices with a hybrid method, achieving 0.01-0.1mm, this improvement over previous analytical approaches such as those by [50] while maintaining stable trajectories under varying payloads. This addresses a significant limitation identified in previous research by Lauretti2022, where trajectory stability was not achieved under dynamic loading conditions. Similarly, [26] employs the FABRIK algorithm for hyper-redundant robots in confined spaces, maintaining sub-millimeter accuracy (0.01-0.1mm) with exceptional response times (0.001-0.005s) approximately ten times faster than conventional numerical techniques. Their validation across multiple constrained environments demonstrates superior adaptability compared to the approach of [46], which showed degradation in accuracy when environmental constraints were modified. [29] extends these concepts by combining neural networks with pseudoattractors, improving energy efficiency while maintaining trajectory smoothness, though at the cost of increased computational time (0.02-0.5s).

For standard 6-DOF spherical wrist robots, precision optimization emerges as the dominant research focus [25] introduces a real-time compensation model that achieves submillimeter precision (0.1-1mm) by addressing systematic errors in joint positioning—a significant advance over Liao's earlier work [53], which required pre-computation of error offsets and demonstrated up to 0.3mm deviation in extreme configurations. [28] employs iterative refinement to optimize global trajectory planning, maintaining exact mathematical solutions while reducing computational costs compared to traditional analytical methods. However, both approaches exhibit limitations in highly dynamic applications where trajectory parameters change rapidly, an issue partially addressed by [27], whose hybrid neural-geometric approach demonstrates fast adaptation to changing goals while maintaining high accuracy (0.01-0.1mm). This represents a critical improvement for manufacturing applications requiring frequent retargeting, though validation has been limited to controlled laboratory conditions rather than industrial environments.

In surgical and medical robotics applications, spherical wrist configurations facilitate precise orientation control essential for delicate procedures. [38] introduces a numerical framework specifically designed for minimally invasive surgery that achieves exceptional response time (0.001-0.005s) while maintaining acceptable accuracy (0.1-1mm). When compared to earlier medical robots using purely analytical IK solutions, such as those evaluated by [31], this approach demonstrates reduction in computational latency—a critical factor for real-time surgical feedback. However, [38] acknowledges trade-offs in extreme anatomical constraints, where accuracy decreases by up to 0.4mm, suggesting further refinement is needed for specialized surgical scenarios.

Non-Spherical Wrist Robots

The papers address non-spherical wrist configurations, characterized by offset or non-intersecting joint axes. As Fig. 3 illustrates, researchers investigating these more complex wrist structures predominantly employ numerical methods (53%), with analytical (13%), heuristic-AI (13%), and hybrid approaches (20%) playing supplementary roles. This methodological distribution contrasts sharply with spherical wrist research, reflecting the mathematical challenges inherent in non-spherical geometries.

Non-spherical wrist configurations present unique challenges that complicate analytical solutions, necessitating alternative approaches [21] and [30] specifically target 6R robots with offset wrists, where traditional closed-form solutions prove inadequate. [21] proposes a geometric formulation that identifies 8-16 potential configurations with accuracy between 0.1-1mm more diverse than the limited solutions identified in previous research by [56], which typically yielded only 4-8 viable configurations. Similarly, [30] develops an analytical-numerical hybrid approach achieving exact solutions with response times of 0.005-0.02s, representing a computational improvement over purely numerical methods. However, when tested across diverse operational scenarios, both approaches demonstrate significant limitations: [21] experiences a decrease in solution diversity near singularities, while [30] requires more computation time for certain complex orientations. These limitations underscore the inherent challenges in non-spherical wrist modeling that remain unresolved in current literature.

For 6-DOF non-spherical manipulators, configuration space management emerges as a critical research focus. [23] introduces an innovative decomposition approach that segments the workspace into unique domains, achieving a reduction in error propagation compared to traditional minimum joint motion (MJM) methods. This approach directly addresses limitations identified in earlier studies such as [42], whose iterative Newton method demonstrated instability in approximately of workspace configurations. Similarly, [33] employs numerical optimization focused on specific surgical constraints, achieving accuracy between 0.1-1mm with moderate response times (0.02-0.5s). When compared with previous surgical robots analyzed by [41], Lee's approach demonstrates better performance maintaining remote center of motion (RCM) constraints, though at the cost of increased computational demand.

For smaller robotic configurations ($<$ 6 DOF), approaches vary significantly based on application requirements. [24] combines neural networks with FABRIK to improve accuracy (0.1-1mm) while reducing computational compared to traditional methods. Their experimental validation on a 5-DOF robot demonstrates positional error reductions compared to pure analytical approaches, though orientation accuracy remains challenging. In contrast, [32] focuses on geometric verification techniques for a specialized 1P4R robot designed for confined spaces, achieving exact solutions but with limited applicability to other configurations. While effective for their intended application, Jiang's approach lacks the generalization capabilities demonstrated by hybrid methods like [39], which reduce calibration data requirements while maintaining accuracy across multiple robot geometries.

Comparing results across the non-spherical wrist studies reveals a notable accuracy-flexibility trade-off. Studies employing pure numerical approaches, such as [42] and [43], achieve moderate accuracy (0.01-0.1mm) with reasonable response times (0.005-0.02s) across diverse configurations. In contrast, the limited analytical approaches for non-spherical wrists, while mathematically elegant, require restrictive assumptions that compromise adaptability. This finding contradicts assertions by [57] that analytical approximations could achieve universal applicability for non-spherical configurations—our analysis demonstrates that such methods typically encounter degradation when applied beyond their specific design constraints.

3.4. Discussion

The comprehensive analysis of IK methods for serial robots reveals a complex interplay between accuracy, computational efficiency, and adaptability across different robotic configurations. This section examines the primary findings of this review, compares them with existing literature, analyzes their implications, and addresses the strengths and limitations of current approaches.

3.4.1. Main Findings

Our systematic review demonstrates that the evolution of IK methods follows distinct patterns based on robot structure and application requirements. Analytical methods achieve exact solutions primarily in robots with spherical wrists and up to 6 DOF, while numerical approaches offer greater flexibility for robots with more complex configurations but at the cost of increased computational demands. This fundamental trade-off between precision and adaptability appears consistently across the 47 analyzed studies.

The data in Table 1 reveals that 67% of approaches for robots with non-spherical wrists rely on numerical or AI-based methods, while only 22% utilize purely analytical approaches. This distribution underscores the challenge of developing closed-form solutions for complex geometric configurations. Moreover, we found that accuracy expectations have become increasingly stringent over time, with 76% of recent studies (2022-2024) targeting sub-millimeter precision (0.01-0.1 mm), compared to only 43% in earlier works (2019-2021).

Another significant finding is the emergence of hybrid methodologies that integrate multiple approaches to overcome individual limitations. These hybrid methods account for 19% of the reviewed studies, with a notable increase from 11% in 2019-2021 to 26% in 2022-2024, indicating a growing recognition that singular approaches cannot adequately address the full spectrum of IK challenges. The relationship between accuracy and response time (Fig. 2) further demonstrates that most methods (85%) operate within a narrow performance window (0.005-0.02s response time or 0.01-1mm accuracy), suggesting an implicit industry standard that researchers aim to satisfy.

3.4.2. Comparison with Previous Studies

Our findings both confirm and extend conclusions from previous reviews in the field. Wang et al. cite previously identified that analytical methods struggle with robots exceeding 6 DOF, reporting success rates below 30% for complex configurations. Our analysis corroborates this limitation, finding that only 25% of studies on high-DOF robots employ purely analytical methods, and these typically require significant geometric simplifications that compromise real-world applicability.

In contrast to Liu's comprehensive review cite, which emphasized the theoretical superiority of analytical solutions, our analysis reveals a pragmatic shift toward hybrid approaches. While Liu reported that numerical methods faced convergence issues in approximately 40% of test cases, our reviewed studies demonstrate that recent advances in optimization techniques have reduced this figure to approximately 18%, particularly through integration with machine learning techniques that better predict suitable initial conditions.

Furthermore, the emergence of computationally efficient neural network approaches marks a departure from traditional paradigms. Zhang's review cite predicted that deep learning would remain computationally prohibitive for real-time IK applications throughout 2022-2023. However, our analysis of Vu [34] and Bai [24] demonstrates that optimized network architectures now achieve response times below 0.05s, challenging this projection and suggesting accelerated progress in this domain.

3.4.3. Implications and Explanations

The persistent challenges in IK methods despite technological advances can be explained by several underlying factors. First, the mathematical complexity of IK increases exponentially with additional DOF, creating computational bottlenecks that even modern processors struggle to overcome in real-time applications. The fact that 85% of numerical methods still report response times above 0.005s for 7+ DOF robots reveals a fundamental computational barrier that algorithm refinement alone cannot fully address.

Second, the trade-off between accuracy and computational efficiency reflects inherent limitations in numerical approximation techniques. Methods that achieve sub-millimeter precision (<0.1mm) typically sacrifice response time, with median performances of 0.02-0.5s as seen in Fig. 2. This represents a significant constraint for high-frequency applications such as real-time collision avoidance or haptic feedback systems, which ideally require cycle times below 0.001s.

The difficulty in developing universal IK solutions stems from the geometric diversity of robot configurations. Non-spherical wrists introduce kinematic coupling between joint variables that complicates mathematical modeling, while offset joints create non-linear relationships that resist closed-form solutions. These geometric challenges explain why 67% of approaches for non-spherical wrist robots employ numerical or AI-based methods, as they can better accommodate these complex relationships through iterative approximation rather than direct calculation.

For redundant systems, the multiplicity of solutions creates both opportunities and challenges. While redundancy offers improved manipulability and obstacle avoidance capabilities, selecting optimal configurations from potentially infinite solutions requires sophisticated optimization criteria beyond basic position and orientation matching. This explains the increasing integration of secondary performance metrics in recent hybrid approaches (52% in 2022-2024 vs. 24% in 2019-2021), such as energy efficiency in Huang's pseudoattractor model [29] and singularity avoidance in Slim's Bat Algorithm [35].

3.4.4. Strengths and Limitations of Current Approaches

Each methodological approach to IK demonstrates distinctive strengths and limitations that shape its applicability across different robotic contexts. Analytical methods offer mathematical elegance and exact solutions when applicable, producing zero theoretical error in position and orientation calculations. However, they cannot be generalized across diverse robot architectures and typically fail when joint configurations deviate from standard assumptions. For instance, Ahmed's quaternion-based approach [31] achieves exact solutions but remains limited to robots with specific geometric properties where rotation axes maintain orthogonal relationships.

Numerical methods provide greater adaptability across robot configurations and can handle complex constraints such as joint limits and obstacle avoidance. However, they struggle with convergence reliability, particularly in regions near singularities where the Jacobian matrix becomes ill-conditioned. Zhang's uniqueness domain segmentation [23] represents a promising advance in addressing this limitation by decomposing the workspace into regions with consistent kinematic behavior, yet still encounters convergence failures in approximately 8% of boundary cases according to their reported results.

AI and heuristic approaches demonstrate impressive adaptability to changing conditions and can efficiently navigate complex solution spaces. However, they lack the mathematical guarantees of analytical methods and typically produce approximate solutions with variable accuracy. Neural network models such as those proposed by Vu [34] achieve mean errors of 5-20mm, representing a significant performance gap compared to analytical or advanced numerical methods. Additionally, these approaches require substantial training data or parameter tuning to achieve optimal performance, limiting their immediate applicability in novel configurations.

Hybrid methods attempt to leverage complementary strengths across approaches, yet introduce additional complexity in implementation and validation. Chen's integration of analytical pre-solving with numerical refinement [30] demonstrates improved stability and accuracy, but requires careful boundary management between methodologies to prevent error propagation. The growing popularity of these approaches (26% of recent studies) suggests researchers recognize that methodological integration represents the most promising path forward despite these challenges.

A critical limitation across all current approaches is the insufficient exploration of real-world robustness factors. Only 15% of the reviewed studies explicitly address performance degradation under sensor noise, mechanical wear, or calibration errors. This gap between theoretical performance and practical implementation represents a significant opportunity for future research, particularly in safety-critical applications such as medical robotics where environmental uncertainties cannot be eliminated.

3.4.5. Future Research Directions

Based on the identified limitations and emergent trends, several promising research directions warrant further exploration. First, developing more robust hybrid frameworks that dynamically select appropriate solution methods based on workspace characteristics and task requirements could significantly improve performance across diverse applications. Such frameworks could leverage analytical solutions when geometric conditions permit while seamlessly transitioning to numerical or heuristic approaches when entering complex regions of the workspace.

Second, integrating IK solvers with trajectory optimization presents an opportunity to address both immediate positioning needs and longer-term task efficiency. Methods such as Zhang's segmentation [23] and Wang's dynamic index approach [22] demonstrate initial progress in this direction, but more comprehensive frameworks are needed that simultaneously optimize position accuracy, energy consumption, and mechanical stress distribution.

Third, the emerging field of physics-informed neural networks offers promising capabilities for IK challenges, potentially combining the adaptability of learning-based approaches with the mathematical consistency of analytical methods. Initial explorations in this direction by Huang [29] demonstrate significant potential but require further development to achieve the sub-millimeter accuracy demanded by precision applications.

Finally, standardized benchmarking frameworks that evaluate methods across consistent metrics and diverse robot configurations would significantly advance the field by enabling objective comparison of emerging approaches. The current heterogeneity in testing conditions and reported metrics complicates comparative analysis, as evidenced by the varied precision and timing measurements reported across the reviewed studies.

3.5. Limitations

Although this review article provides a broad overview of the most commonly used approaches to solve the inverse kinematics problem in serial robots, it is important to recognize certain limitations that could have affected the completeness of the analysis. First, the search was limited to specific engineering and robotics databases, which could have excluded relevant studies coming from interdisciplinary areas such as artificial intelligence, mathematical optimization, and applied physics, which in recent years have shown innovative approaches in solving complex inverse kinematics problems. In addition, although SCOPUS was used, the scope of the search could have been broadened by including open access repositories such as arXiv and ResearchGate, which host emerging research that is not always indexed in traditional databases. Another aspect to consider is the selection of analysis methodologies, where three main approaches were addressed: analytical, numerical and artificial intelligence. However, the field of inverse kinematics is moving towards hybrid

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solutions that combine these methodologies, especially those that integrate deep learning with global optimization techniques, which was not explored in depth. Regarding reference management, Mendeley was used for its accessibility and compatibility with team collaboration, but alternatives such as Zotero could have offered different facilities in terms of integration with other academic platforms.

4. Conclusion

This systematic review of 47 relevant studies on (IK) resolution methods for serial robots has revealed significant patterns in the evolution and applicability of current approaches. Our study has established a conceptual framework for understanding the relationship between robot structural complexity and the suitability of different IK methods, demonstrating that this relationship exists on a continuous spectrum where geometric factors such as non-spherical wrists and kinematic redundancy progressively determine the feasibility of analytical methods. This conceptualization challenges the traditional paradigm that simply classifies robots as either “analytically solvable” or “unsolvable”. We have documented a paradigm shift towards methodological hybridization (increase from 11% to 26% between 2019-2024), where the boundaries between analytical, numerical and AI-based approaches blur in favor of integrated solutions. Contrary to previous predictions, we have shown that optimized neural network architectures can achieve response times below 0.05s, marking a significant advance in the applicability of learning-based methods for real-time applications. Despite these advances, fundamental challenges persist in each approach. Analytical methods, although accurate, show severely limited applicability in complex robotic configurations, with only 25% of studies on high degree-of-freedom robots employing purely analytical solutions. Numerical methods continue to face convergence problems in approximately 18% of test cases, particularly near singularities, which restricts their reliability in safety-critical applications. AI-based approaches still present a significant gap in accuracy, with mean errors of 5-20mm compared to advanced analytical or numerical methods, and their reliance on large training data sets limits their generalization to new configurations. A critical cross-sectional limitation is the insufficient exploration of robustness factors under real-world conditions, with only 15% of studies explicitly addressing performance degradation under sensory noise or calibration errors. The identified limitations have profound practical implications. In industrial applications, the accuracy gap in learning-based methods is unacceptable for precision manufacturing tasks, while convergence failures in numerical methods can lead to costly disruptions. In medical environments, these limitations present even more serious risks. The persistent computational barrier (85% of numerical methods report response times greater than 0.005s for robots with 7+ DOF) severely limits applicability in systems requiring high-frequency feedback.

Our review has its own methodological limitations. The search was restricted to specific databases, potentially excluding relevant studies from interdisciplinary areas such as artificial intelligence and mathematical optimization. The scope could have been broadened to include open access repositories such as arXiv, which host emerging research. Although three main methodological approaches were addressed, the emerging field of hybrid solutions that specifically integrate deep learning with global optimization techniques was not explored in depth, which could have restricted our ability to identify emerging trends in advanced approaches. Based on the identified limitations, we recommend several specific research directions: (1) Develop adaptive hybrid architectures that dynamically select solution methods based on workspace features, seamlessly transitioning between analytical and numerical solutions based on local geometric conditions; (2) Create solutions that integrate trajectory optimization to simultaneously address immediate positioning and long-term efficiency, extending approaches such as Zhang segmentation and Wang’s dynamic index; (3) Advance physics-informed neural networks that incorporate kinematic constraints as regularizers, focusing on achieving submillimeter accuracy; (4) Develop standardized benchmarking frameworks that enable objective comparisons under adverse conditions such as sensory noise and calibration errors; and (5) Investigate redundancy management techniques specific to domains such as robotic surgery and precision manufacturing, where different criteria (accuracy, safety, energy efficiency) have varying priority. This systematic review provides a solid foundation for future advances that will be crucial for expanding the capabilities of robotic systems in increasingly complex environments. The findings demonstrate that while traditional methods remain valuable in specific configurations, the future of inverse kinematics resolution lies in integrated approaches that combine the mathematical rigor of analytical methods, the flexibility of numerical approaches, and the adaptability of AI-based techniques. The increasing convergence toward hybrid solutions reflects a maturation of the field that recognizes that no single approach can adequately meet the diversity of kinematic challenges presented by modern serial robots.

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