

# Design and Quality Evaluation of the Position and Attitude Control System for 6-DOF UAV Quadcopter Using Heuristic PID Tuning Methods

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## ABSTRACT

Nowadays, UAV quadcopters are widely used in many fields, specially in transporting the lightweight goods parcels. This article aims to design and evaluation of the quality of the 6-DOF UAV quadcopter control system using heuristic PID tuning methods to ensure stable control of flight position and attitude. Firstly, the article presents the dynamic mathematical model of the 6-DOF UAV quadcopter, including 3 Euler angle variables and 3 flight position and altitude variables. From there, the article proposes the 6-DOF UAV control system structure with two single control loops for flight attitude, yaw angle and two dual control loops for roll-pitch angles, flight position. And then, the article presents the application of the heuristic PID tuning methods to each control loop of a 6-DOF UAV quadcopter to calculate the PID controller parameters to ensure stable control the desired flight position and altitude. The simulation results and evaluating the 6-DOF UAV quadcopter control system quality in Matlab, using the proposed heuristic PID controllers, show that the PID controllers according to the Tyreus-Luyben method gives the best quality, with a steady-state error of less than 1%. The main contribution of this article is the comparative analysis of three heuristic PID tuning methods – Ziegler-Nichols, Tyreus-Luyben, PID tuner – for controlling the position and attitude of a 6-DOF UAV quadcopter. These findings demonstrate that the proposed PID controllers can be effectively implemented in practical UAV applications, enhancing the stability and performance of quadcopters in various fields.

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## 1. Introduction

Nowadays, UAVs (Unmanned Aerial Vehicle) are applied in many areas of life, industry and national defense and security, such as transporting goods, monitoring construction works, tracking targets, searching and rescuing. UAVs provide efficient and accurate goods delivery through advanced UAV control systems. Among them, UAV quadcopters are characterized by four symmetrically arranged propellers, driven by four electric motors, capable of flexible control, so quadcopter UAVs are widely used in many fields, specially in transporting the lightweight goods parcels. For goods transportation systems, UAVs always require stable and accurate control during the performance of transportation tasks from basic ones such as take-off, landing, altitude control, flight position control,

flight trajectory control to complex tasks such as obstacle avoidance, flight route optimization. However, UAV quadcopter is the complex structured equipment system, consisting of many interconnected components. This is also an object with strong nonlinearity, operating in harsh large space environments. Therefore, the research on UAV quadcopters always attracts a lot number of scientists and science-technology enterprises.

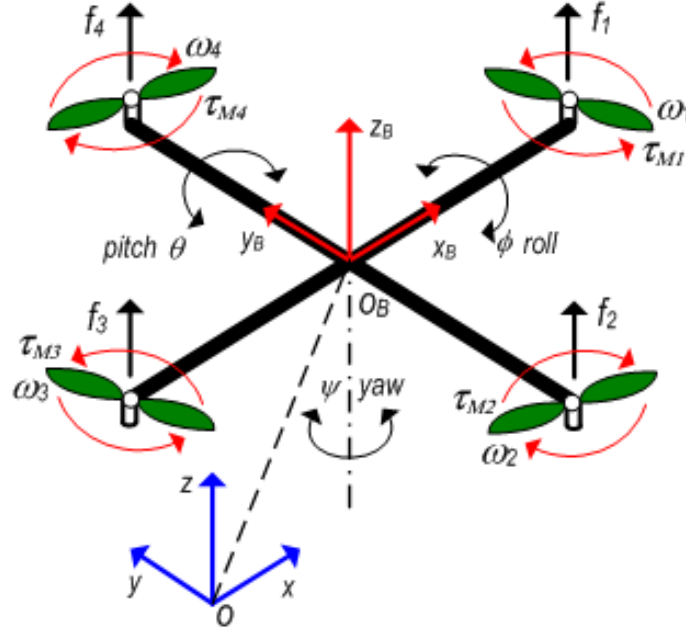
Research works [1], [2] present the kinematic and dynamic model of UAV and design the basic controllers, meeting the requirements of basic flight position control. The works [3], [4] proposed the backstepping control for UAV considering the nonlinear object and achieving certain results. The works [5], [6] introduced fuzzy logic control for UAV in the presence of external disturbances and model uncertainties and achieves good results. And there are many studies based on the proportional-integral-derivative (PID) controller for UAV that have achieved certain results [7]-[9], [14], [16], [18]-[21], [25]-[28], [31]-[32], [35]-[36], [42]-[44], [46]-[49]. These studies have exploited the advantages of PID control and applied UAV control to achieve certain results. Furthermore, another research works on UAV control have also been presented. Furthermore, many other research works on UAV control have also been proposed, such as linear quadratic regulator (LQR) [10], model predictive control (MPC) [11], linearization feedback control [12], sliding mode control (SMC) [13], [39]-[40], optimal control [15], [22], optimized PID control [17], [34], nonlinear robust control [23], advanced PID control [24], neuron network based PID control [30], [41], nonlinear control [33], adaptive PID control [45], PID control integrated state feedback [50], extended Kalman filter and adaptive fuzzy PID control [51], fuzzy-PID control [52]-[53]. It can be seen that the research works on UAV control using PID control law are very popular because of its simplicity and efficiency. These studies use the linear mathematical model of UAV, then design the PID controller, develop the PID controller combined with advanced control algorithms, such as fuzzy logic, neural network, optimization or use MPC, SMC, LQR control methods and have brought about certain control results, generally meeting the requirements for position control and balance of UAV. However, the UAV control algorithms published in these studies are difficult to fully access, the UAV model is considered in many different aspects, adjusting the parameters of the UAV controllers in practical implementation is difficult.

This article presents the dynamic mathematical model of 6-degree of freedom (DOF) UAV quadcopter, then designs and adjusts the parameters of 6-PID controllers for 6-DOF UAV quadcopter, meeting the requirements of flight attitude and position stable control. The 6-PID controllers for the 6-DOF UAV quadcopter are synthesized based on the heuristic PID tuning methods, due to its simplicity, efficiency and good control performance. The main contribution of this article is the comparative analysis of three heuristic PID tuning methods – Ziegler-Nichols, Tyreus-Luyben, Matlab PID tuner toolbox – for controlling the position and attitude of a 6-DOF UAV quadcopter. This shows that the proposed PID controllers can be effectively implemented in practical UAV applications, enhancing the stability and performance of 6-DOF UAV quadcopters in various fields, specially in transporting the lightweight goods parcels. In addition to the content presented above, the structure of the rest parts of the article includes: the mathematical dynamic model of the 6-DOF UAV quadcopter – part 2; The design of the position and attitude control system for 6-DOF UAV quadcopter – part 3; The simulation and quality evaluation of the position and attitude control system for 6-DOF UAV quadcopter – part 4; finally – conclusions and future research.

## 2. Dynamic Mathematical Models of 6-DOF UAV Quadcopter

The structural diagram of the UAV quadcopter and the coordinate systems used in building the dynamic mathematical model of the UAV quadcopter are shown in Fig. 1. In Fig. 1, the UAV's angular velocity, torque, and the corresponding force generated by the four propellers, driven by four electrical motors from 1 to 4 (also called rotors). The dynamic mathematical model of the 6-DOF quadcopter is built using the following assumptions: i) The quadcopter model is a homogeneous and symmetric block. ii) Mathematical model built on a cross-shaped quadcopter, the center of the quadcopter is the center of gravity; iii) Neglecting elasticity, consider the transmission system absolutely rigid.

The UAV's absolute translational position in the inertial frame  $x, y, z$ -axes, attached to the ground, is denoted  $\xi$ ,  $\xi = [x \ y \ z]^T$ . The angular position is determined by the Euler angles,  $\eta, \eta = [\phi \ \theta \ \psi]^T$ . The roll angle  $\phi$  determines the rotation of the UAV quadcopter around the  $x$ -axis. The pitch angle  $\theta$  determines the rotation of the UAV quadcopter around the  $y$ -axis. The yaw angle  $\psi$  determines the rotation of the UAV quadcopter the  $z$ -axis. The vector  $\varepsilon$  is combination of the translational position vector  $\xi$  and the angular position vector  $\eta$ ,  $\varepsilon = [\xi \ \eta]$



**Fig. 1.** The UAV quadcopter structure in the body frame and ground coordinates

The UAV's body frame has the origin point that is the UAV's center of mass point. In the UAV's body frame, translational velocities are denoted by  $V_B$  and angular velocities by  $v$ .

$$V_B = [v_{x,B} \ v_{y,B} \ v_{z,B}]^T, v = [p \ q \ r]^T \quad (1)$$

Applying Euler-Lagrange method to the UAV quadcopter with external forces and torques, we have the following equation form [9], [13], [20], [21].

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\varepsilon}} \right) - \frac{\partial L}{\partial \varepsilon} \quad (2)$$

Where,  $\tau$  denotes the rotational torques, and  $f$  is the translational forces.

The Lagrange function  $L$  is the sum of rotational energy  $E_{rot}$  and translational energy  $E_{trans}$  minus potential energy  $E_{pot}$  [9], [13], [20].

$$L = E_{trans} + E_{rot} - E_{pot} = \frac{m}{2} \dot{\xi}^T \dot{\xi} + \frac{1}{2} v^T I v - mgz \quad (3)$$

The overall dynamic mathematical model of UAV quadcopter is divided into translational motion and rotational motion by considering the corresponding state vectors. The translational and rotational components of UAV quadcopter are independent [13], [22], so they can be studied separately. The translational Euler-Lagrange equation are represented as follow.

$$f = RF_B = m\ddot{\xi} + mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{F}{m} \begin{bmatrix} C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi \\ C_\phi C_\theta \end{bmatrix} \quad (5)$$

Where  $C_\alpha = \cos(\alpha)$ ;  $S_\alpha = \sin(\alpha)$ ;  $\alpha = \phi, \theta, \psi$

The translational motion of UAV quadcopter can be represented as follows [22], [23].

$$\begin{aligned} m\ddot{x} + A_x\dot{x} &= F(C_\psi S_\theta C_\phi + S_\psi S_\phi) \\ m\ddot{y} + A_y\dot{y} &= F(S_\psi S_\theta C_\phi - C_\psi S_\phi) \\ m\ddot{z} + mg + A_z\dot{z} &= F(C_\phi C_\theta) \end{aligned} \quad (6)$$

Where  $F$  is the thrust force acting along the z-axis.  $A_x, A_y, A_z$  are the coefficients of air resistance in the corresponding directions of the axes of the inertial frame.

The UAV quadcopter's translational dynamic equation is presented as follow.

$$\begin{aligned} \ddot{x} &= -\frac{A_x\dot{x}}{m} + (C_\psi S_\theta C_\phi + S_\psi S_\phi) \frac{F}{m} \\ \ddot{y} &= -\frac{A_y\dot{y}}{m} + (S_\psi S_\theta C_\phi - C_\psi S_\phi) \frac{F}{m} \\ \ddot{z} &= -\frac{A_z\dot{z}}{m} - g + (C_\phi C_\theta) \frac{F}{m} \end{aligned} \quad (7)$$

In addition, the Jacobian matrix  $J(\eta)$  from angular velocities  $v$  to angular velocities  $\dot{\eta}$  is defined as follows [13], [21], [22].

$$J(\eta) = J = W_\eta^T I W_\eta$$

$$J = \begin{bmatrix} I_{xx} & 0 & -I_{xx}S_\theta \\ 0 & I_{yy}C_\phi^2 + I_{zz}S_\phi^2 & (I_{yy} - I_{zz})C_\phi S_\phi C_\theta \\ -I_{xx}S_\theta & (I_{yy} - I_{zz})C_\phi S_\phi C_\theta & I_{xx}S_\theta^2 + I_{yy}S_\phi^2 C_\theta^2 + I_{zz}C_\phi^2 C_\theta^2 \end{bmatrix} \quad (8)$$

The rotational energy can be represented in the inertial frame as follow [13], [21], [22].

$$E_{rot} = \frac{1}{2} v^T I v = \frac{1}{2} \dot{\eta}^T J \dot{\eta} \quad (9)$$

Therefore, the Euler-Lagrange equation with the external angular force - torques of the UAV rotors are represented as follow.

$$\tau = \tau_B = J \ddot{\eta} + \frac{d}{dt}(J) \dot{\eta} - \frac{1}{2} \frac{\partial}{\partial \eta} (\dot{\eta}^T J \dot{\eta}) = J \ddot{\eta} + C(\eta, \dot{\eta}) \dot{\eta} \quad (10)$$

Where, the matrix  $C(\eta, \dot{\eta})$  is Coriolis term, containing the gyros and centripetal terms.

Thefore, the UAV quadcopter's rotation dynamic equation is shown as follow.

$$\ddot{\eta} = J^{-1} [\tau_B - C(\eta, \dot{\eta}) \dot{\eta}] \quad (11)$$

$$C(\eta, \dot{\eta}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \quad (12)$$

$$C_{11} = 0;$$

$$C_{12} = (I_{yy} - I_{zz})(\dot{\theta} C_\phi S_\phi + \dot{\psi} S_\phi^2 C_\theta) + (I_{zz} - I_{yy})\dot{\psi} C_\phi^2 C_\theta - I_{xx}\dot{\psi} C_\theta;$$

$$C_{13} = (I_{zz} - I_{yy})\dot{\psi} C_\phi S_\phi C_\theta^2$$

$$\begin{aligned}
C_{21} &= (I_{zz} - I_{yy})(\dot{\theta}C_\phi S_\phi + \dot{\psi}S_\phi C_\theta) + (I_{yy} - I_{zz})\dot{\psi}C_\phi^2 C_\theta + I_{xx}\dot{\psi}C_\theta \\
C_{22} &= (I_{zz} - I_{yy})\dot{\phi}C_\phi S_\phi; \\
C_{23} &= -I_{xx}\dot{\psi}S_\theta C_\theta + I_{yy}\dot{\psi}S_\phi^2 S_\theta C_\theta + I_{zz}\dot{\psi}C_\phi^2 S_\theta C_\theta \\
C_{31} &= (I_{yy} - I_{zz})\dot{\psi}C_\phi^2 S_\phi C_\phi - I_{xx}\dot{\theta}C_\theta \\
C_{32} &= (I_{zz} - I_{yy})(\dot{\theta}C_\phi S_\phi S_\theta + \dot{\phi}S_\phi^2 C_\theta) + (I_{yy} - I_{zz})\dot{\phi}C_\phi^2 C_\theta + I_{xx}\dot{\psi}S_\theta C_\theta - I_{yy}\dot{\psi}S_\phi^2 S_\theta C_\theta - I_{zz}\dot{\psi}C_\phi^2 S_\theta C_\theta \\
C_{33} &= (I_{yy} - I_{zz})\dot{\phi}C_\phi S_\phi C_\theta^2 - I_{yy}\dot{\theta}S_\phi^2 C_\theta S_\theta - I_{zz}\dot{\theta}C_\phi^2 C_\theta S_\theta + I_{xx}\dot{\theta}C_\theta S_\theta
\end{aligned}$$

The rotation matrix from the body frame to the inertial frame is determined  $R$ , [13]-[16], [36], [37].

$$R = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\theta & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\theta & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \quad (13)$$

Where  $S_\alpha = \sin \alpha$ ,  $C_\alpha = \cos \alpha$ ,  $\alpha = \phi, \theta, \psi$ . The rotation matrix  $R$  is orthogonal, so that  $R^{-1} = R^T$  which is the rotation matrix from the inertial frame to the body frame.

The matrix that transforms the angular velocity from the inertial frame to the body frame is  $W_\eta$  and from the body frame to the inertial frame is  $W_\eta^{-1}$  [9], [13], [17].

$$v = W_\eta \dot{\eta} \quad \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (14)$$

$$\dot{\eta} = W_\eta^{-1} v \quad \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & S_\phi / C_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (15)$$

Where,  $T_\theta = \tan \theta$ ;  $W_\eta$  is reversible in the case of  $\theta \neq (2k-1)\frac{\phi}{2}$ ,  $k = 1, 2, \dots$

The UAV quadcopter has a symmetrical structure with four arms aligned with the body x-axis and y-axis. Thus, the inertia matrix is diagonal matrix  $I$ , is defined as follow.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (16)$$

The angular velocity of the  $i$ -rotor, denoted by  $\omega_i$ , generated the lift fore  $f_i$  in the direction of the rotor axis. In addition, the angular velocity and acceleration also generate the torque  $\tau_{Mi}$ , around the rotor shaft [13], [18].

$$f_i = k\omega_i^2, \tau_{Mi} = b\omega_i^2 + I_M \dot{\omega}_i \simeq b\omega_i^2 \quad (17)$$

Where,  $k$  is the lift force coefficient,  $b$  is the torque coefficient of the rotor, and  $I_M$  is the moment of inertia of the rotor. Since the propeller is so light, the  $\dot{\omega}_i$  effect is often omitted.

The principle of controlling 4 UAV rotors is as follows: The roll  $\phi$  angle adjustment is achieved by increasing the 4<sup>th</sup> rotor angular velocity and decreasing the 2<sup>nd</sup> rotor angular velocity. The pitch  $\theta$  angle adjustment is achieved by increasing the 3<sup>rd</sup> rotor angular velocity and decreasing the 1<sup>st</sup> rotor angular velocity. The yaw  $\psi$  angle adjustment is achieved by increasing the angular velocities of two opposite 1,3 rotors and decreasing the angular velocities of two other 2,4 rotors. Therefore, the combination of the four lift rotors forces create the  $F$  thrust force in direction of the body along z-axis. Total torque  $\tau_B$  consists of torques  $\tau_\phi, \tau_\theta, \tau_\psi$  in the direction of the corresponding body frame angles [13], [19].

$$F = \sum_{i=1}^4 f_i = k \sum_{i=1}^4 \omega_i^2, F_B = \begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix} \quad (18)$$

$$\tau_B = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} l k (\omega_4^2 - \omega_2^2) \\ l k (\omega_3^2 - \omega_1^2) \\ b(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{bmatrix} \quad (19)$$

Where  $l$  is the distance from the UAV's centre to the propeller.

Thus, we obtain the dynamic mathematical model of the 6-DOF UAV quadcopter represented by equations (7) and (11), which correspond to the translational dynamic part and the rotational dynamic part of the 6-DOF UAV quadcopter, and they are closely related to each other through the transformation matrix. Based on the 6-DOF UAV quadcopter dynamic mathematical model, the article presents the 6-DOF UAV quadcopter position and attitude control system structure, and then the heuristic PID controllers design for the 6-DOF UAV quadcopter is presented in the following section.

### 3. Design of the Position and Attitude Control System for 6-DOF UAV Quadcopter

The 6-DOF UAV quadcopter has four controlled rotors, but there are six state variables, position  $\xi$  and angular  $\eta$ . Equations (7), (11) present the dynamic model of the 6-DOF UAV quadcopter, denoting the interaction between state variables and the total thrust force  $f$  and torque  $\tau$  generated by the rotors. The UAV quadcopter is kept the position and attitude by the total thrust force  $F$ , which also affects acceleration along the z-axis. The roll angle acceleration powered by the torque  $\tau_\phi$ , the pitch angle acceleration powered by the torque  $\tau_\theta$ , and the yaw angle acceleration is powered by the torque  $\tau_\psi$  [14], [32]-[34]. The block diagram of the position and attitude control system for 6-DOF UAV quadcopter is presented in Fig. 2.

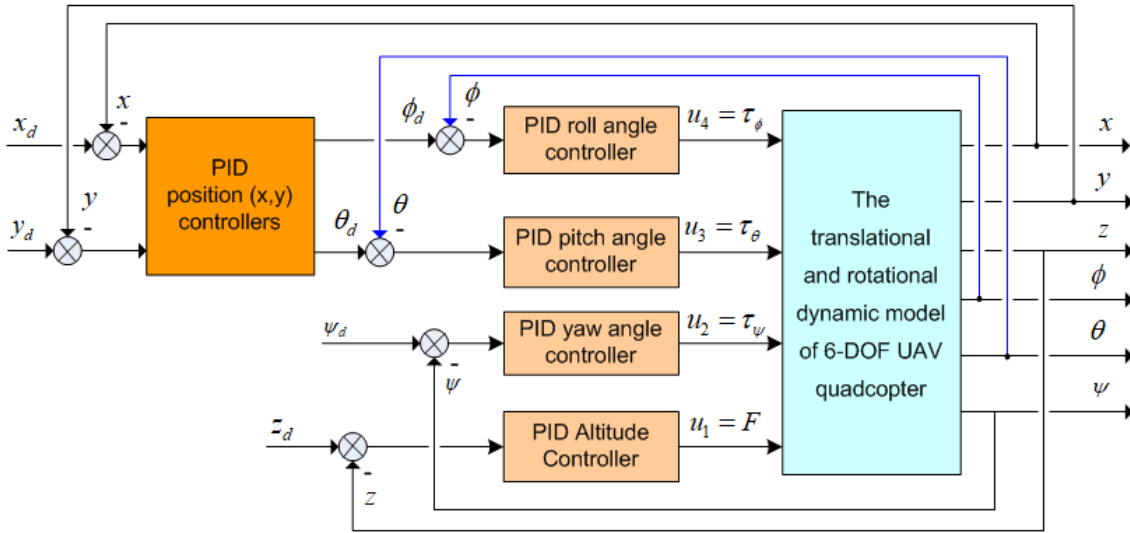


Fig. 2. The position and attitude control system for 6-DOF UAV quadcopter

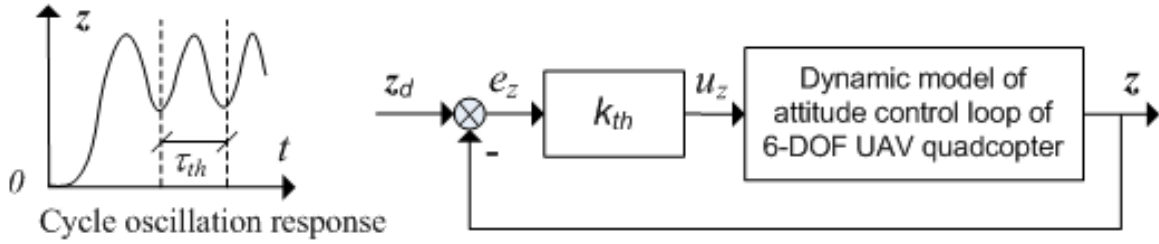
The position and attitude control system for 6-DOF UAV quadcopter consist of the two single-feedback control loops: attitude loop  $z$  and yaw angle loop  $\psi$ ; and two cascade feedback control loops: roll and pitch angles feedback control loops – inner control loops, and outer control loops –  $x, y$  position feedback control loops [9].

Here, the PID controller consists of three components, P- proportion, I- integral and D- derivation, described in standard form as follow [14], [25]-[28], [41].

$$u(t) = k_p \left( e(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau + T_D \frac{de(t)}{dt} \right) = k_p e(t) + k_I \int_0^t e(\tau) d\tau + k_D \frac{de(t)}{dt} \quad (20)$$

Where,  $e(t)$  is the controlled state error,  $u(t)$  is the control signal; the corresponding PID parameters are  $k_p$ - proportional coefficient,  $k_I$ ,  $T_I$ - integral coefficient,  $k_D$ ,  $T_D$ - derivative coefficient.

Currently, there are many methods to determine the parameters  $k_p$ ,  $k_I$ ,  $k_D$  for the PID controller. Among them, the heuristic methods of the Tyreus-Luyben [29], [30] and the Ziegler-Nichols [29], [31] allow easy determination of PID parameters based on the control object model. The general principle of these heuristic methods is to use the proportional P- component and then adjust the value of  $k_p$  until the control system response achieves the cyclic oscillation, as Fig. 3.



**Fig. 3.** Diagram of heuristic principle to determine PID parameters

The synthesis of PID controllers for 6-DOF UAV quadcopter, using the heuristic methods of the Tyreus-Luyben or the Ziegler-Nichols, based on observing the closed-loop cycle oscillation response with each control loop at  $k_{th}$  and  $t_{th}$  values, and is generalized as follows:

Phase 1. Perform experiments on the attitude control loop ( $z$ ), as shown in Fig. 3, by adjusting  $k_p$  to the  $k_{th}$  value, corresponding to the cycle oscillation object output response, and then determine the oscillation cycle period  $t_{th}$ .

Phase 2. Calculate the PID parameters for the attitude control loop ( $k_{Pz}$ ,  $k_{Iz}$ ,  $k_{Dz}$ ), according to formulas (21) when applying Ziegler-Nichols method [29], [31], or formulas (22) when applying Tyreus-Luyben method [29], [30].

$$k_{Pz} = \frac{3k_{thz}}{5}; k_{Iz} = \frac{2k_{Pz}}{t_{thz}}; k_{Dz} = \frac{k_{Pz}t_{thz}}{8} \quad (21)$$

$$k_{Pz} = \frac{5k_{thz}}{11}; k_{Iz} = \frac{5k_{Pz}}{11t_{thz}}; k_{Dz} = \frac{10k_{Pz}t_{thz}}{63} \quad (22)$$

Phase 3. Continue to perform experiments with the remaining control loops of 6-DOF UAV quadcopter, including  $\phi$ -angle control loop,  $\theta$ -angle control loop,  $\psi$ -angle control loop, and x,y-position control loops, and then determine appropriate PID controller parameters.

### 3.1. Design of the PID Rotation Angles and Attitude Controllers for 6-DOF UAV Quadcopter

The PID rotation angles and attitude controllers calculate the total thrust force and rotor torques for the UAV quadcopter, which are defined by formulas as shown follow.

$$\begin{aligned} \tau_\phi &= (k_{P\phi}(\phi_d - \phi) + k_{I\phi} \int_0^t (\phi_d - \phi) d\tau + k_{D\phi}(\dot{\phi}_d - \dot{\phi}))I_{xx} \\ \tau_\theta &= (k_{P\theta}(\theta_d - \theta) + k_{I\theta} \int_0^t (\theta_d - \theta) d\tau + k_{D\theta}(\dot{\theta}_d - \dot{\theta}))I_{yy} \end{aligned} \quad (23)$$

$$\tau_\psi = (k_{P\psi}(\psi_d - \psi) + k_{I\psi} \int_0^t (\psi_d - \psi) d\tau + k_{D\psi}(\dot{\psi}_d - \dot{\psi}))I_{zz}$$

Where,  $g$  is the gravity,  $m$  is the mass and  $I_{xx}$ ,  $I_{yy}$ ,  $I_{zz}$  is inertia moment of the UAV quadcopter

Applying the Ziegler-Nichols method [29], [31], the PID controllers' parameters for the angles  $(\phi, \theta, \psi)$  control loops, and the altitude ( $z$ ) control loop of the UAV quadcopter are determined by formulas as follow.

$$\begin{aligned} k_{P\phi} &= \frac{3k_{th\phi}}{5}; k_{I\phi} = \frac{2k_{P\phi}}{\tau_{th\phi}}; k_{D\phi} = \frac{k_{P\phi}\tau_{th\phi}}{8} \\ k_{P\theta} &= \frac{3k_{th\theta}}{5}; k_{I\theta} = \frac{2k_{P\theta}}{\tau_{th\theta}}; k_{D\theta} = \frac{k_{P\theta}\tau_{th\theta}}{8} \\ k_{P\psi} &= \frac{3k_{th\psi}}{5}; k_{I\psi} = \frac{2k_{P\psi}}{\tau_{th\psi}}; k_{D\psi} = \frac{k_{P\psi}\tau_{th\psi}}{8} \\ k_{Pz} &= \frac{3k_{thz}}{5}; k_{Iz} = \frac{2k_{Pz}}{\tau_{thz}}; k_{Dz} = \frac{k_{Pz}\tau_{thz}}{8} \end{aligned} \quad (24)$$

Applying the Tyreus-Luyben method [29], [30], the PID controllers' parameters for the angles  $(\phi, \theta, \psi)$  control loops, and the altitude ( $z$ ) control loop of the UAV quadcopter are determined by formulas as follow.

$$\begin{aligned} k_{P\phi} &= \frac{5k_{th\phi}}{11}; k_{I\phi} = \frac{5k_{P\phi}}{11\tau_{th\phi}}; k_{D\phi} = \frac{10k_{P\phi}\tau_{th\phi}}{63} \\ k_{P\theta} &= \frac{5k_{th\theta}}{11}; k_{I\theta} = \frac{5k_{P\theta}}{11\tau_{th\theta}}; k_{D\theta} = \frac{10k_{P\theta}\tau_{th\theta}}{63} \\ k_{P\psi} &= \frac{5k_{th\psi}}{11}; k_{I\psi} = \frac{5k_{P\psi}}{11\tau_{th\psi}}; k_{D\psi} = \frac{10k_{P\psi}\tau_{th\psi}}{63} \\ k_{Pz} &= \frac{5k_{thz}}{11}; k_{Iz} = \frac{5k_{Pz}}{11\tau_{thz}}; k_{Dz} = \frac{10k_{Pz}\tau_{thz}}{63} \end{aligned} \quad (25)$$

### 3.2. Design of the PID Position Controllers for 6-DOF UAV Quadcopter

The position and attitude control system for 6-DOF UAV quadcopter has four inner feedback control loops, which are three rotation angles loops and altitude loop. Two outer feedback loops are performed to adjust the x,y positions of the UAVquadcopter. The desired roll and pitch angles are the output of the outer control loops, and they serve as the input of the inner control loops, respectively, the desired angle  $\phi_d$ ,  $\theta_d$ , (Fig. 2)

When the UAV quadcopter is stable in the space, the roll angle  $\phi$  and pitch angle  $\theta$  have small values. Therefore, by using small angle assumptions,  $S_{\phi_d} \equiv \phi_d$ ,  $S_{\theta_d} \equiv \theta_d$ ,  $C_{\phi_d} = C_{\theta_d} = 1$ , the dynamic equations for the x, y positions loops are simplified in equations as follow [35], [36].

$$\begin{aligned} \ddot{x} &= \frac{u_1}{m}(\theta_d C_\psi + \phi_d S_\psi) \\ \ddot{y} &= \frac{u_1}{m}(\theta_d S_\psi - \phi_d C_\psi) \end{aligned} \quad (26)$$

With  $u_1$  is the UAV quadcopter altitude control signal

At this time, the PID altitude controller is selected as follow.

$$u_1 = k_{Pz}(z_d - z) + k_{Iz} \int_0^t (z_d - z) d\tau + k_{Dz}(\dot{z}_d - \dot{z}) \quad (27)$$

Equation (26) is written as a matrix as follows.

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \frac{u_1}{m} \begin{bmatrix} S_\psi & C_\psi \\ -C_\psi & S_\psi \end{bmatrix} \begin{bmatrix} \phi_d \\ \theta_d \end{bmatrix} \quad (28)$$

Therefore, the pitch and roll desired angles are determined by the formulas.

$$\begin{aligned} \phi_d &= (u_x S_\psi - u_y C_\psi) \\ \theta_d &= (u_x C_\psi + u_y S_\psi) \end{aligned} \quad (29)$$

Where,  $u_x$  and  $u_y$  are control signals, based on the PID control law.

$$\begin{aligned} u_x &= k_{Px}(x_d - x) + k_{Ix} \int_0^t (x_d - x) d\tau + k_{Dx}(\dot{x}_d - \dot{x}) \\ u_y &= k_{Py}(y_d - y) + k_{Iy} \int_0^t (y_d - y) d\tau + k_{Dy}(\dot{y}_d - \dot{y}) \end{aligned} \quad (30)$$

Applying the Ziegler-Nichols method [29], [31], the PID controller parameters for the x, y-position control loops are determined by formulas as shown follow.

$$\begin{aligned} k_{Px} &= \frac{2k_{thx}}{5}; k_{Ix} = \frac{2k_{Px}}{\tau_{thx}}; k_{Dx} = \frac{k_{Px}\tau_{thx}}{8} \\ k_{Py} &= \frac{2k_{thy}}{5}; k_{Iy} = \frac{2k_{Py}}{\tau_{thy}}; k_{Dy} = \frac{k_{Py}\tau_{thy}}{8} \end{aligned} \quad (31)$$

Applying the Tyreus-Luyben method [29], [30], the PID controller parameters for the x, y-position control loops are determined by formulas as shown follow.

$$\begin{aligned} k_{Px} &= \frac{5k_{thx}}{11}; k_{Ix} = \frac{5k_{Px}}{11\tau_{thx}}; k_{Dx} = \frac{10k_{Px}\tau_{thx}}{63} \\ k_{Py} &= \frac{5k_{thy}}{11}; k_{Iy} = \frac{5k_{Py}}{11\tau_{thy}}; k_{Dy} = \frac{10k_{Py}\tau_{thy}}{63} \end{aligned} \quad (32)$$

## 4. Simulation and Quality Evaluation of the Position and Attitude Control System for 6-DOF UAV Quadcopter

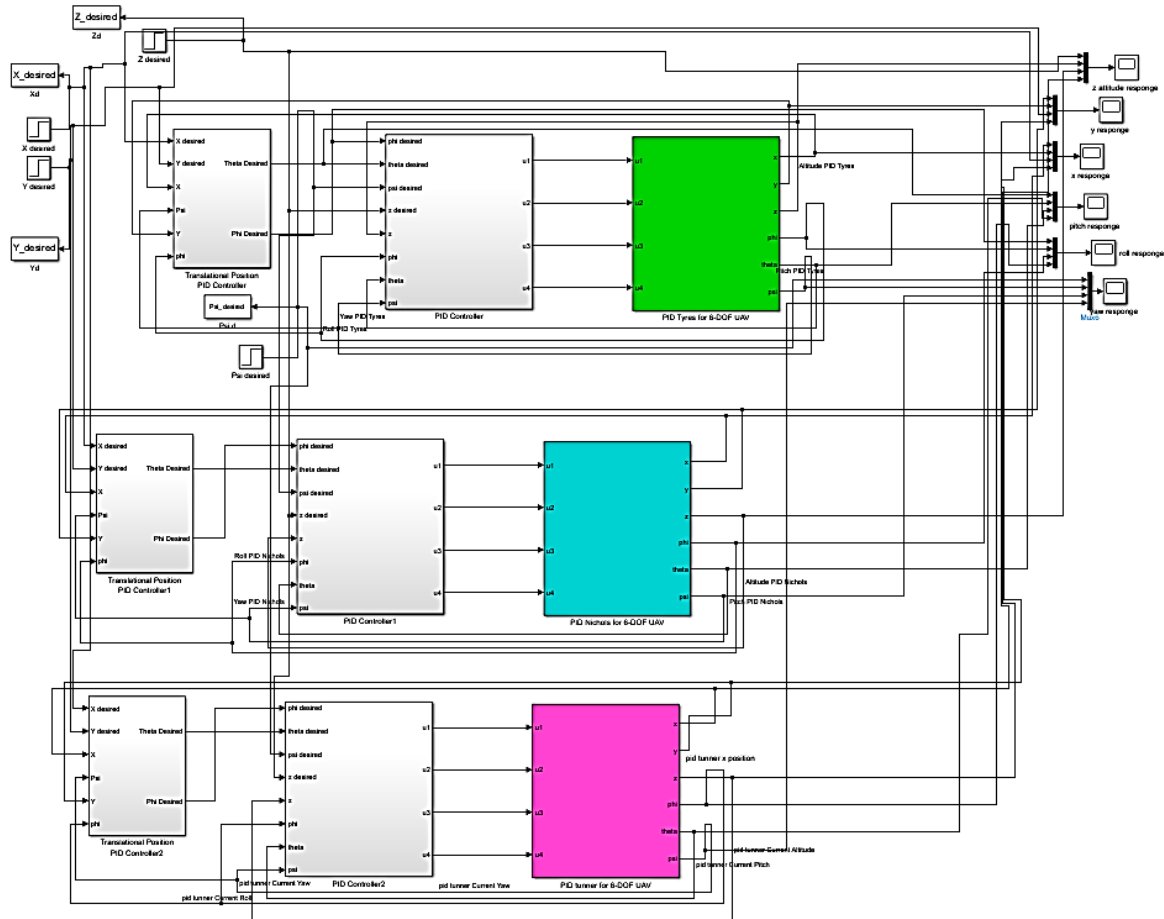
### 4.1. Modeling for the Position and Attitude Control System for 6-DOF UAV Quadcopter

The parameters of the 6-DOF UAV quadcopter are presented as shown in Table 1. The simulation diagram of the position and attitude control system for 6-DOF UAV quadcopter is built in Matlab – Simulink, using three heuristic PID control methods (Tyreus-Luyben, Ziegler-Nichols, Matlab PID tuner), as shown in Fig. 4.

**Table 1.** Parameters of the UAV quadcopter

| Parameter                   | Symbol           | Value                       |
|-----------------------------|------------------|-----------------------------|
| Quad. mass                  | $m$              | 0.516 kg                    |
| Arm length                  | $l$              | 0.225 m                     |
| Gravity                     | $g$              | 9.81 m/s <sup>2</sup>       |
| Inertia moment of the rotor | $I_M$            | 3.368e-5 kg.m <sup>2</sup>  |
| Thrust factor of rotor      | $k$              | 2.996e-6 N.s <sup>2</sup>   |
| Drag coeffi.                | $b$              | 1.260e-7 N.m.s <sup>2</sup> |
| Inertial constants          | $I_{xx}, I_{yy}$ | 4.984e-3 kg.m <sup>2</sup>  |
|                             | $I_{zz}$         | 8.958e-3 kg.m <sup>2</sup>  |

The position and attitude control system for 6-DOF UAV quadcopter consists of three subsystems corresponding to the position and attitude control subsystem using PID controllers according to three heuristic methods: Tyreus-Luyben, Ziegler-Nichols, Matlab PID tuner. Each position and attitude control subsystem includes three main blocks: (i). The UAV quadrotor dynamic model block is built based on equations (1)-(29). (ii). The attitude and rotation angulars control block calculates the thrust force and rotor torques for the UAV quadcopter, based on PID control laws, using formulas (24) with Ziegler-Nichols method or using formulas (26) with Tyreus-Luyben method. (iii). The x,y translational position control block calculates the desired roll and pitch angles, based on PID laws, using formulas (31) with Ziegler-Nichols method or using formulas (32) with Tyreus-Luyben method.



**Fig. 4.** The 6-DOF UAV quadcopter control system using proposed heuristic PID controllers: Tyreus-Luyben, PID Ziegler-Nichols, PID tuner

#### 4.2. The PID Parameters Determination Experiment for 6-DOF UAV Quadcopter

Applying the Ziegler-Nichols method or Tyreus-Luyben method, as above mention, we can determine the parameters of six PID controllers for the 6-DOF UAV quadcopter with each the control loop:  $\phi$ -roll angle control loop,  $\theta$ -pitch angle control loop,  $\psi$ -yaw angle control loop, z-attitude control loop, and x,y-position control loops.

Fig. 5 presents the heuristic results using Tyreus-Luyben method for the z-attitude control loop. We obtain the z- attitude response with cycle oscillations at the value  $k_{th}=124.99$  and oscillation period  $t_{th}=3.52s$ . Applying the formula (25), we calculate the PID parameters for the z-attitude control loop, using the Tyreus-Luyben method, as follow  $k_{pz} = 56.8136, k_{Iz} = 7.3365, k_{Dz} = 31.7435$ .

Implementing similar experiments with the rest control loops of 6-DOF UAV quadcopter control system, we calculate the parameters of the PID controllers for controlling the 6-DOF UAV quadcopter, using the Tyreus-Luyben method, as shown in Table 2.

Calculating similarly with Ziegler-Nichols method, we determine the parameters of the PID controllers for controlling the 6-DOF UAV quadcopter, as shown in Table 3.

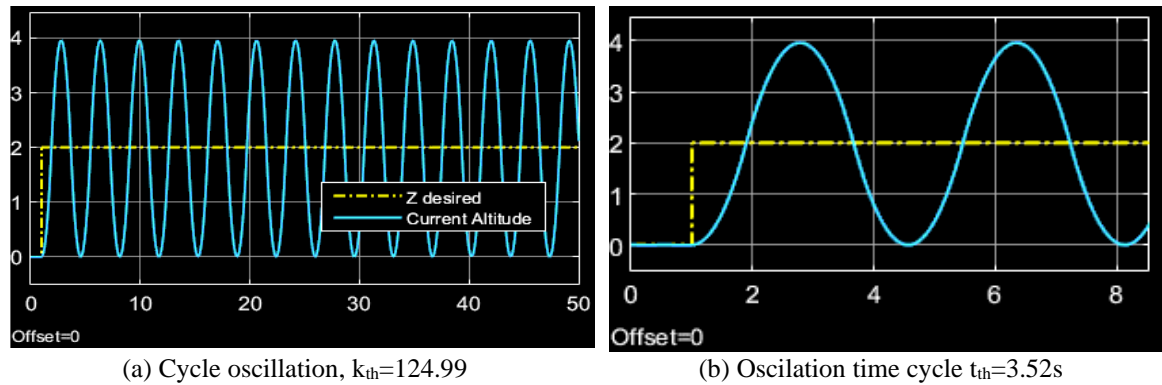


Fig. 5. Cycle oscillation response of z-yaw attitude control loop

Table 2. Parameters of PID controllers for 6-DOF UAV quadcopter, applying Tyreus-Luyben method

| PID parameters | Roll Controller<br>( $\phi$ ) | Pitch Controller<br>( $\theta$ ) | Yaw Controller<br>( $\psi$ ) | Position Controller<br>(x) | Position Controller<br>(y) | Altitude Controller<br>(z) |
|----------------|-------------------------------|----------------------------------|------------------------------|----------------------------|----------------------------|----------------------------|
| $k_p$          | 1.5909                        | 1.8180                           | 0.6090                       | 0.0031                     | 0.0031                     | 56.8136                    |
| $k_i$          | 0.001                         | 0.0012                           | 0.0015                       | 0.00007                    | 0.00000095                 | 7.3365                     |
| $k_d$          | 182.6091                      | 198.7208                         | 17.8394                      | 0.0099                     | 0.7299                     | 31.7435                    |

Table 3. Parameters of PID controllers for 6-DOF UAV quadcopter, using Ziegler-Nichols method

| PID parameters | Roll Controller<br>( $\phi$ ) | Pitch Controller<br>( $\theta$ ) | Yaw Controller<br>( $\psi$ ) | Position Controller<br>(x) | Position Controller<br>(y) | Altitude Controller<br>(z) |
|----------------|-------------------------------|----------------------------------|------------------------------|----------------------------|----------------------------|----------------------------|
| $k_p$          | 2.1201                        | 2.4010                           | 0.8041                       | 0.0048                     | 0.0048                     | 74.9940                    |
| $k_i$          | 0.8411                        | 0.501                            | 0.0020                       | 0.00000008                 | 0.00000009                 | 42.6102                    |
| $k_d$          | 1.3109                        | 28.7665                          | 80.4000                      | 72.0000                    | 64.0000                    | 32.9974                    |

In addition, after building the 6-DOF UAV quadcopter model in Matlab, we can still use Matlab PID tuner toolbox to adjust the PID parameters of the 6-DOF UAV quadcopter position and attitude control system, as shown in Table 4.

Table 4. Parameters of PID controllers for 6-DOF UAV quadcopter, using Matlab PID tuner

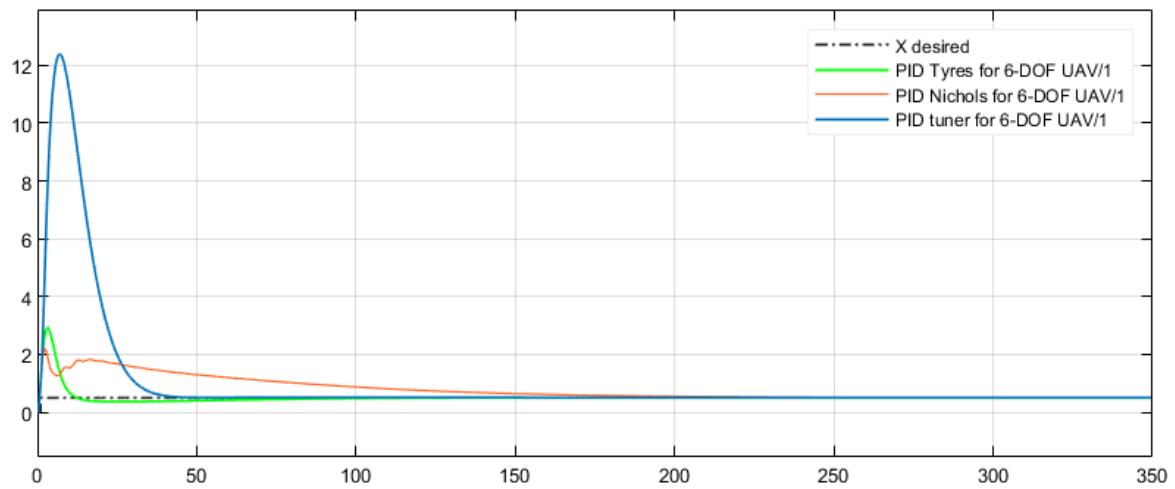
| PID parameters | Roll Controller<br>( $\phi$ ) | Pitch Controller<br>( $\theta$ ) | Yaw Controller<br>( $\psi$ ) | Position Controller<br>(x) | Position Controller<br>(y) | Altitude Controller<br>(z) |
|----------------|-------------------------------|----------------------------------|------------------------------|----------------------------|----------------------------|----------------------------|
| $k_p$          | 53.006                        | 51.1051                          | 52.8964                      | 0.0034708                  | 0.0034708                  | 57.005                     |
| $k_i$          | 0.041                         | 0.0398                           | 0.03685                      | 0.0000004                  | 0.00000001                 | 0.001                      |
| $k_d$          | 26.205                        | 26.3251                          | 26.1984                      | 0.035105                   | 0.0401021                  | 23.1020                    |

Matlab PID tuner toolbox is capable of fast single loop PID tuning, easily implemented with Simulink PID controller blocks. Matlab PID tuner allows simple tuning of PID controller parameters to achieve stable designs with good response times through vivid visual observation of response.

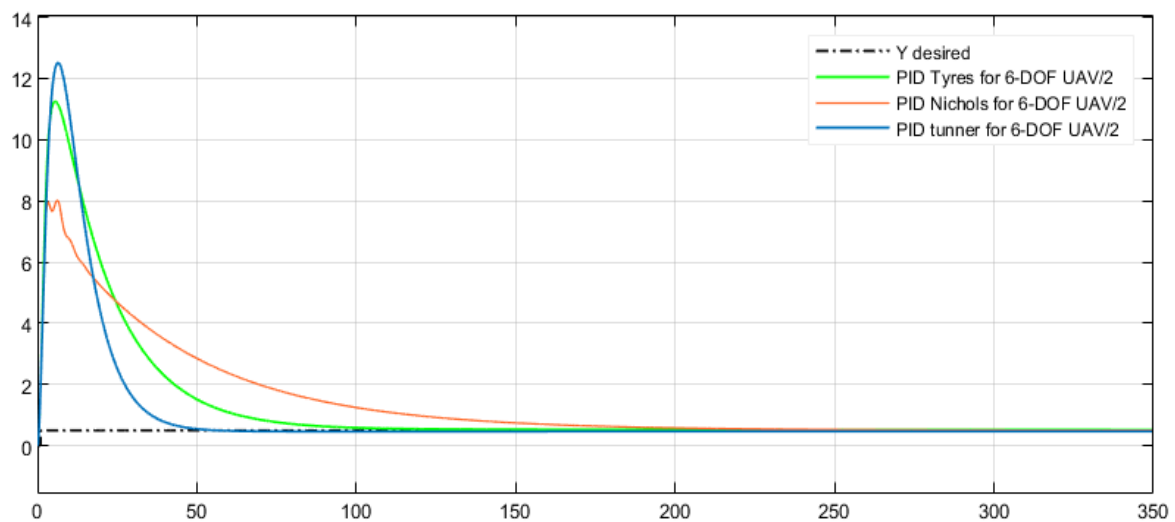
#### 4.3. Simulation Results of the Position and Attitude Control System for 6-DOF UAV Quadcopter

The desired values for simulating the 6-DOF UAV quadcopter:  $x_d=0.5m$ ,  $y_d=0.5m$ ,  $z_d=2.0m$ ,  $\phi_d=0$ ,  $\theta_d=0$ ,  $\psi_d=1rad/s$ .

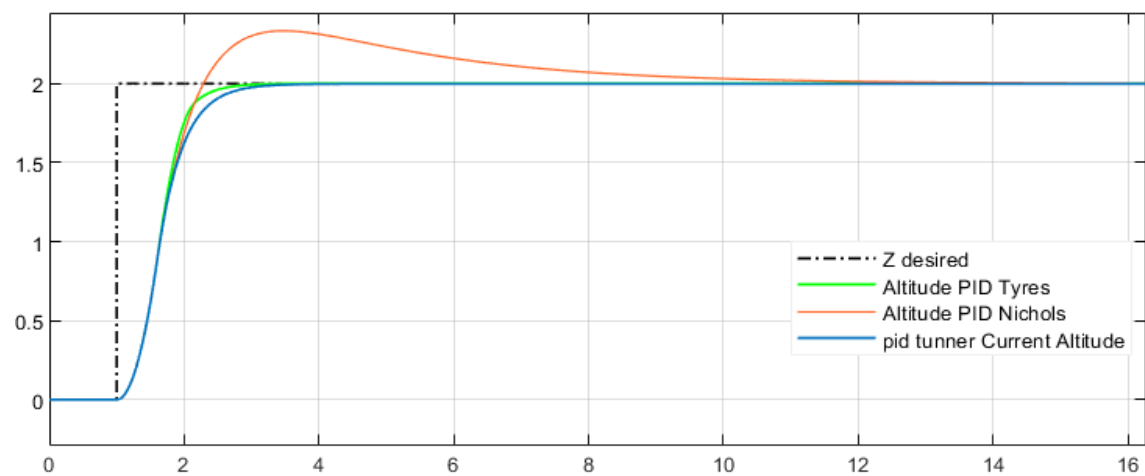
Simulation results of the position and attitude control system for 6-DOF UAV quadcopter with the above three proposed PID controllers are shown in Fig. 6, Fig. 7, Fig. 8, Fig. 9, Fig. 10, Fig. 11, Fig. 12.



**Fig. 6.** The  $x$ -position response of UAV quadcopter



**Fig. 7.** The  $y$ -position response of UAV quadcopter



**Fig. 8.** The  $z$ -altitude response of the UAV quadcopter

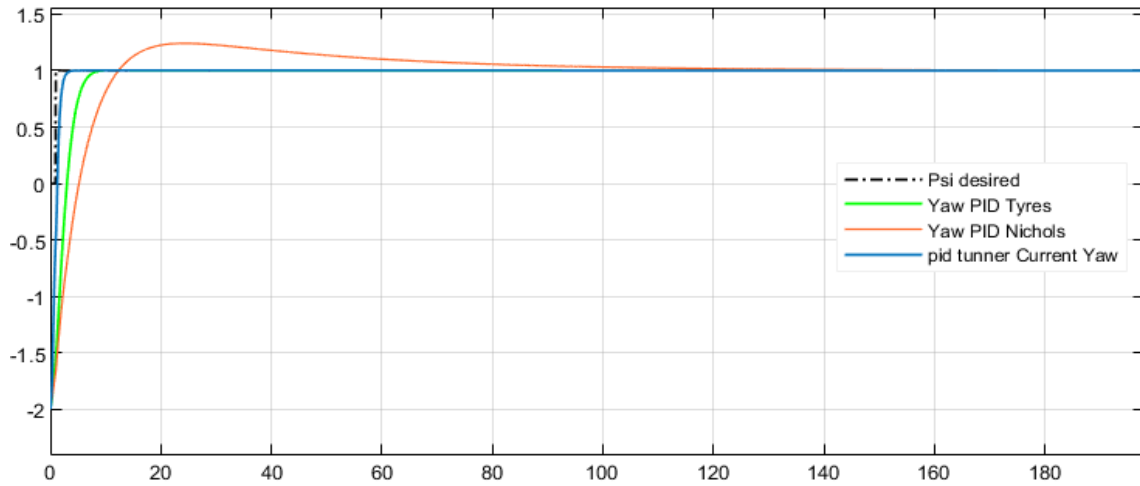


Fig. 9. The  $\psi$ -yaw response of the UAV quadcopter

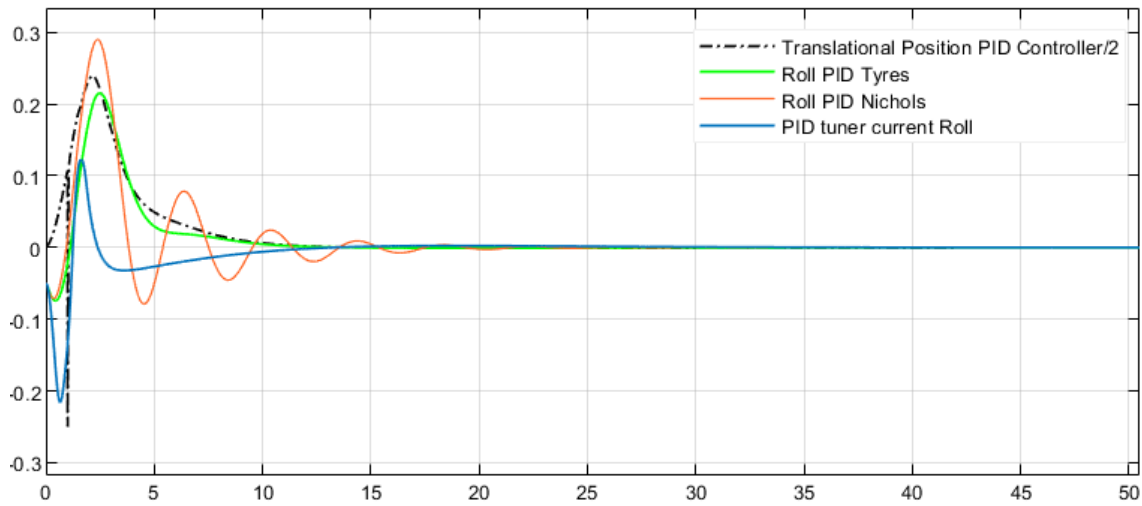


Fig. 10. The  $\phi$ -roll response of the 6-DOF UAV quadcopter

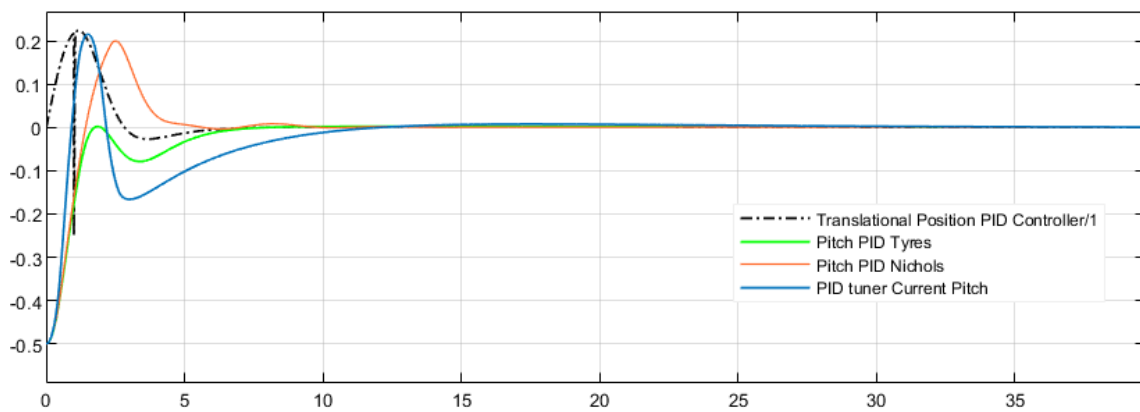
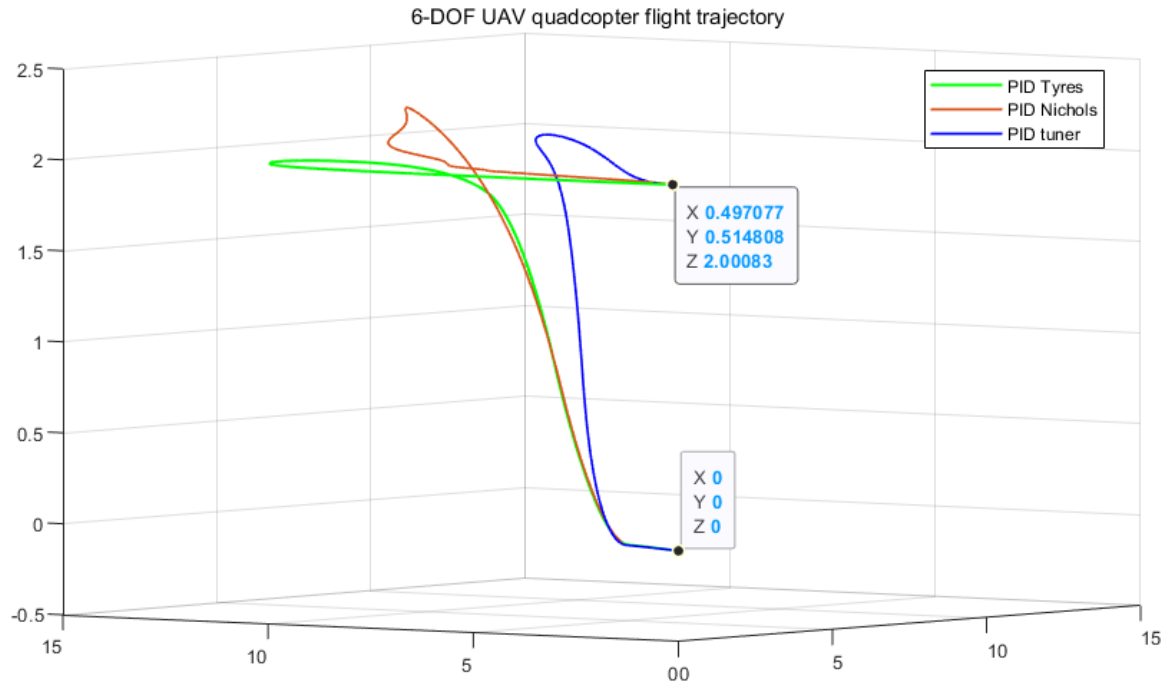


Fig. 11. The  $\theta$ -pitch response of the 6-DOF UAV quadcopter

The simulation results show that the position and attitude control system for 6-DOF UAV quadcopter with six proposed heuristic PID controllers, for  $z$ -altitude,  $\psi$ -angle,  $\phi$ -angle,  $\theta$ -angle and  $x, y$ -position control loops, ensures the flight trajectory stability with the steady-state error

approximate 0, rise time from 1 second to less than 10 seconds, steady time from 5 second to less than 350 seconds.



**Fig. 12.** The 6-DOF UAV quadcopter flight trajectory in 3D x,y,z-axes

The position and attitude responses of the 6-DOF UAV quadcopter in 3D space are introduced in Fig.12. It shows the UAV quadcopter flies fast to reach the desired altitude. And then, the UAV quadcopter is stabilized by the roll, pitch angle controllers and continues to adjust its flight direction, by the yaw angle controller, go to the x,y- desired position by the x,y-position controllers.

The quality indexes of the UAV quadcopter position and attitude control system using the proposed heuristic PID controllers are determined and shown in Table 5, Table 6.

**Table 5.** The quality indexes of the 6-DOF UAV quadcopter control system using PID Tyreus-Luyben

| Controller<br>Quality index | PID $\psi$ -angle | PID $\phi$ -angle | PID $\theta$ -angle | PID x-positions | PID y-positions | PID z-altitude |
|-----------------------------|-------------------|-------------------|---------------------|-----------------|-----------------|----------------|
| Rise time                   | Small, <6s        | Small, <5s        | Small, <5s          | Small, <5s      | Small, <5s      | Small, <1s     |
| Steady time                 | Small, <8s        | Normal, <10s      | Small, <10s         | Normal, <15s    | Large, <100s    | Small, <5s     |
| Overshoot                   | Small, <5%        | Small, <25%       | Small, <10%         | Normal, <30%    | Large           | Small, <5%     |
| Steady-state error          | Approx. 0         | Approx. 0         | Approx. 0           | Very small, <1% | Very small, <1% | Approx. 0      |

**Table 6.** The quality indexes of the 6-DOF UAV quadcopter control system using PID Ziegler-Nichols

| Controller<br>Quality index | PID $\psi$ -angle | PID $\phi$ -angle        | PID $\theta$ -angle | PID x-positions | PID y-positions | PID z-altitude |
|-----------------------------|-------------------|--------------------------|---------------------|-----------------|-----------------|----------------|
| Rise time                   | Small, <10s       | Small, <5s               | Small, <5s          | Small, <5s      | Small, <5s      | Small, <1s     |
| Steady time                 | Large, <100s      | Normal, <10s             | Small, <10s         | Large, <100s    | Large, <350s    | Small, <10s    |
| Overshoot                   | Large, <25%       | Normal, <30% oscillation | Normal, <20%        | Normal, <30%    | Large           | Normal, <20%   |
| Steady-state error          | Approx. 0         | Approx. 0                | Approx. 0           | Small, <3%      | Small, <3%      | Approx. 0      |

The data in Table 5 show that the heuristic PID control system using Tyreus-Luyben method quickly controls the UAV quadcopter to stabilize the desired Euler angle position and desired altitude with steady time from 5-10 seconds, steady-state error approximately 0, then stabilizes the x,y position with steady time from 15-100 seconds, steady-state less than 1%. And when applying the Ziegler-Nichols tuning method, in Table 6, UAV quadcopter is controlled to stabilize the desired Euler angle position and desired altitude with steady time from 10-100 seconds, steady-state error approximately 0, then stabilizes the x,y position with steady time from 15-350 seconds, steady-state less than 3%. Comparative analysis of quality indexes of the position and altitude control system of 6-DOF UAV quadcopter with proposed heuristic PID controllers, we found that the heuristic PID controllers using Tyreus-Luyben method give better performance than PID controllers using Ziegler-Nichols method, as shown clearly through the quality indexes: steady time, steady-state error, especially with x,y,z-control loops, as presented in Table 5 and Table 6.

When applying the Matlab PID tuner toolbox, we performed manually the PID parameter adjustment many times, and finally we achieved the responses of 6-DOF UAV quadcopter as shown in Fig. 6, Fig. 7, Fig. 8, Fig. 9, Fig. 10, Fig. 11, Fig. 12. We found that it is very difficult to simultaneously manually adjust the parameters of six PID controllers to obtain the best responses of six UAV quadcopter control loops. In other words, using Matlab PID tuner toolbox to adjust the parameters of PID controller for 6 interleaving control loops of 6-DOF UAV quadcopter is not simple, because the PID tuner is based on single-loop control object linearization and ignores the UAV quadcopter interleaving. This further confirms that the application of the empirical calculation method of Tyreus-Luyben, Ziegler-Nichols is very important, at least as an important initial PID parameters for control, then to improve the control quality of complex control objects, such as 6-DOF UAV quadcopter. With the support of Matlab PID tuner, researchers can quickly tune PID parameters to get the desired response for single-loop control objects. For multi-loop control objects, complex inter-channel control objects, PID tuner allows tuning, observing the visual response of each priority control loop, as desired. As with UAV quadcopter control, it is possible to consider the priority of the UAV quadcopter altitude control loop.

## 5. Conclusion

In this article, the dynamic mathematical model of the 6-DOF UAV quadcopter is developed to control its position and attitude stability. The 6-DOF UAV quadcopter model is divided into 6 sub-models corresponding to six control loops according to UAV quadcopter state variables, including three Euler angles variables and three position and altitude variables. The research in this article also develops the structure of 6-DOF UAV quadcopter control loops, specifically using two single control loops with attitude and yaw angle variables, two double control loops for roll-pitch angles variables and x,y- position variables, where the two inner control loops are roll angle and pitch angle, and the two outer control loops are the x,y- position. Based on this 6-DOF UAV quadcopter PID control system structure, the article focuses on designing PID controllers using the heuristic PID tuning methods, specifically here applying three heuristic methods, which are Tyreus-Luyben method, Ziegler-Nichols method, Matlab PID tuner toolbox.

From the simulation results and the quality assessment tables of the 6-DOF UAV quadcopter control system using the proposed heuristic PID controllers, it shows that the PID controllers according to the Tyreus-Luyben method gives the best quality, with steady-state error less than 1%, steady-time less than 100 seconds. The process of designing PID controllers according to these heuristic methods allow for dynamic adjustment of PID parameters to achieve better 6-DOF UAV quadcopter control system quality, helping to apply these PID controllers in practice more feasible. And it can be seen that the contribution of this study is the comparative analysis of three heuristic PID tuning methods (Ziegler-Nichols, Tyreus-Luyben, Matlab PID tuner toolbox) in the position and attitude control system of 6-DOF UAV quadcopter. The research results show that the proposed heuristic PID controllers can be effectively deployed in practical UAV quadcopter applications, improving the stability and performance of 6-DOF UAV quadcopter in many fields, especially in the

transportation of light parcels. However, it is not simple to tune the parameters of 6 PID controllers simultaneously and manually to achieve good response for controlling 6-DOF UAV quadcopter.

Future research focuses on evaluating the impact of aerodynamic turbulence, comprehensively considering nonlinearity, channel interleaving on UAV quadcopter, further developing intelligent PID control algorithm, sliding mode control algorithm, intelligent control algorithm based on artificial intelligence, and optimizing for position and altitude control of UAV quadcopter.

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**Conflicts of Interest:** The authors declare no conflicts of interest.

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