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Quadrotor Modeling Approaches and Trajectory Tracking Control Algorithms: A Review

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ABSTRACT

Quadrotor unmanned aerial vehicles are utilized in basically every sector of society, including the business, civil, and military industries. Popular applications include delivery, agriculture, target-acquisition, surveying, surveillance, and rescue. They are widely used due to their exceptional features such as accuracy, capability to perform swift inspections, simplicity in deploying perilous and uncertain missions, and additional praiseworthy attributes. This article presents a comprehensive analysis of the theoretical frameworks that have been proposed for the purpose of quadrotor modelling and control. Detailed examinations are conducted on every methodology that underpins the control algorithms, spanning from traditional linear to modern. The analysis looks at hybrid control technique models, which incorporate adaptive components across multiple controllers to improve overall performance and resilience by addressing individual algorithm shortcomings. This analysis also delves deeper into potential future research avenues. These include the development of learning-based or hybrid methodologies that employ machine learning and artificial intelligence to optimize performance and adaptability. For instance, model reference adaptive control systems can learn adaptation laws through machine learning techniques, as opposed to depending on predefined adaptation laws. By training neural networks or fuzzy logic controllers to forecast optimal adaptation parameters based on sensor data, the quadrotor can adjust to fluctuating conditions more effectively. A comparison table is provided to elaborate on the advantages, disadvantages, and hybrid versions of each control algorithm. This will serve as a concise guide that will promote innovation, facilitate the selection and integration of appropriate control algorithms, and enhance the functionality of quadrotor control systems.

1. Introduction

Over the past few years, unmanned aerial vehicles (UAVs) have proliferated in the sky, transforming a variety of industries from agriculture and surveillance to package delivery and disaster relief. Consequently, the use of UAVs is rapidly expanding in nearly all segments of society, including the military, the civilian sector, and the commercial sector [1]. This is mostly due to advancements in sensor technology, communication, and control systems, which enable the production of cheaper, more efficient, and smaller UAVs. There are three types of UAVs: fixed-wing, flapping-wing, and rotorcraft. Fixed-wing UAVs resemble conventional aircraft in their design. These UAVs move through the air with the assistance of their fixed wings, which produce lift. Flapping-wing UAVs use their wings to simulate the flight of insects or birds. In contrast to fixed-wing UAVs, flapping-wing UAVs are propelled by jet engines or propellers and produce thrust and lift by rhythmically flapping their wings. Rotorcraft UAVs generate lift and control flight through the use of vertically oriented rotor blades. These include various multirotor configurations such as helicopters and quadcopters. Fixed-wing aircrafts, unlike rotorcrafts, require a runway for takeoff and landing, whereas rotorcrafts have vertical takeoff and landing capability as well as hovering capability [2], making them the best choice for civilian applications.

Among UAVs, quadrotor unmanned aerial vehicle (QUAV) is a prominent attraction in the aerial robotic sector, particularly in autonomous engineering and research domains. This is owing to its exceptional performance, vertical take-off and landing capability, agility, simple structure, and so on. QUAVs typically come in cross or plus layouts and have four rotors. To balance the net torque produced, two rotors are permitted to rotate in a clockwise direction, and the remaining two are allowed to rotate in a counter-clockwise way. The quadrotor's thrust and torque values, as well as its direction of motion, are determined by the angular speed of each rotor. For instance, a quadrotor can move forward by speeding up the front rotors relative to the back ones; similarly, the difference in speed between the left and right rotors can provide a rolling torque. The quadrotor's autopilot module and the rotor's electronic speed controllers are directly coupled. The autopilot module manages the dynamics of the quadrotor as well as the mixing of rotor speeds for desired movements like rolling, yawing, and so forth. The procedure of controlling quadrotor motion can be divided into two stages: attitude control and position control. The position controller aids in tracking the intended trajectory, whereas the attitude controller maintains the quadrotor's desired orientation. To keep the quadrotor in the desired orientation, the attitude controller compares the commanded roll, pitch, and yaw angles to the quadrotor's actual Euler angles. Then it tries to eliminate any differences between the two measurements. The roll and pitch commands are most likely generated by the position controller since the quadrotor is an underactuated system, in contrast to the heading angle (yaw).Furthermore, in order to enable the quadrotor to follow the intended trajectory, the position controller generates the control signal to balance out any disparity between the desired 3D trajectory and the quadrotor's actual flight trajectory. There are literature reviews available that discuss modelling, the identification of quadrotor models, and supplementary control algorithms. Regrettably, a significant portion of these sources fail to offer indispensable guidance on mathematical concepts that extend beyond the scope of control laws.

This article aims to provide important insight into two areas: first, the most common quadrotor modelling techniques and the assumptions that are typically made when developing such models; and second, the most prevalent quadrotor control algorithms, their upsides and downsides, the assumptions that are typically made when putting these control laws into practice, and the future directions in developing the control algorithm. The ensuing sections go into more detail about the aforementioned goals.

2. Modelling

A quadrotor, as the name suggests, is made up of four rotors. The angular speed of these four rotors controls the direction of motion of the quadrotor, and each spinning rotor creates its own force (F_i) and torque (τ_i). As shown in the Fig. 1, the thrust and torques generated by the front, rear, right, and left rotors are (F_1, τ_1) , (F_3, τ_3), (F_2, τ_2) and (F_4, τ_4) accordingly.

The quadrotor's upward thrust (T_z) is formed by the net sum of individual rotor forces (F_i), whilst the rolling and pitching torques are formed by the proper mixing of the rotor forces, and the

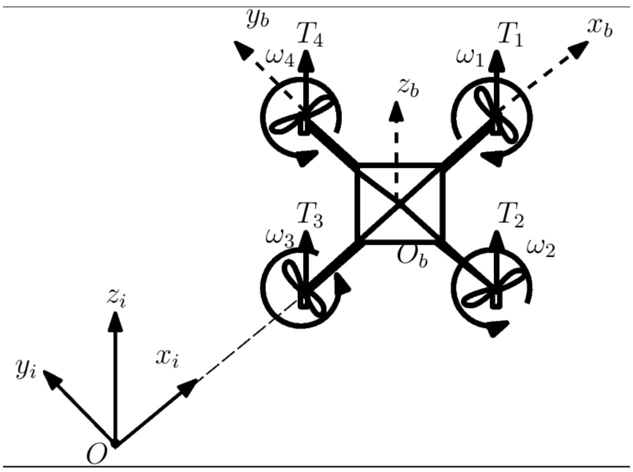

Fig. 1. Schematic diagram of quadrotor

yawing torque is generated by the combination of individual rotor torques as given in [3],

$$\begin{aligned} T_z &= \sum_{i=1}^{4} F_i \\ \tau_\phi &= (F_1 - F_3)l \\ \tau_\theta &= (F_4 - F_2)l \\ \tau_\psi &= \tau_1 + \tau_2 + \tau_3 + \tau_4 \end{aligned} \tag{1}$$

The ensuing motion of a quadrotor in space as a result of these forces and torques is represented using various rotating frames such as inertial frame, body frame, vehicle frame, and so on. Let $\zeta = (x_i,\, y_i,\, z_i)$ and $V = (V_x,\, V_y,\, V_z)$ represent the quadrotor's location and velocity in the inertial frame, whereas $X_b = (x_b,\, y_b,\, z_b)$ and $V_b = (u,\, v,\, w)$ represent those in the body frame. The orientation of the quadrotor is described by Euler angles, ($\eta = (\phi,\, \theta,\, \psi)$). Assume the quadrotor being studied has mass m and moment of inertia I, then the moment of inertia is usually believed to be a diagonal matrix due to its symmetry. The most prevalent quadrotor motion formulations are detailed here.

2.1. Euler-Lagrange Approach

Let's say that a vector q is generated by combining the position and orientation vectors $q = (\zeta, \eta)$. The difference between the quadrotor's potential energy and kinetic energy (including rotational and translational kinetic energy), is defined as the Lagrangian $\mathcal{L}(q, \dot{q})$ as in [4],

$$\begin{aligned} \mathcal{L}(q, \dot{q}) &= KE_{Trans} + KE_{rot} - PE \\ &= \frac{1}{2} m \dot{\zeta}^T \dot{\zeta} + \frac{1}{2} \dot{\eta}^T I \dot{\eta} - mgz \end{aligned} \tag{2}$$

Subsequently, the quadrotor model is expressed in relation to the specified Lagrangian function, as well as the total thrust and torques exerted by the quadrotor as,

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = \begin{bmatrix} T_z \\ \tau \end{bmatrix} \tag{3}$$

where $\tau = (\tau_\phi,\, \tau_\theta,\, \tau_\psi)$ and T_z and τ are determined by the following equations;

$$
\begin{aligned}
T_z &= m\ddot{\zeta} + mg\begin{bmatrix}0\\0\\1\end{bmatrix}\\
\tau &= W_n^T I W_n \ddot{\eta} + C(\eta,\dot{\eta})\dot{\eta}
\end{aligned}
\tag{4}
$$

Here, W_n denotes the transformation matrix, while $C(\eta,\dot{\eta})$ refers to the Coriolis term. Hence, the values of $\ddot{\zeta}$ and $\ddot{\eta}$, which represent the acceleration in translation and rotation respectively, can be derived from (4).

2.2. Newton-Euler Approach

The spatial trajectory of a quadrotor is determined by a multitude of frames, which comprise the inertial frame, body frame, and vehicle frame. To ascertain the system dynamics through the application of Newton's laws of motion, frame transformations are required, given that these laws are only applicable in an inertial frame. When the velocity of the quadrotor is denoted as V, it is possible to formulate Newton's law for translational motion with a thrust force T and the angular velocity of the airframe relative to the inertial frame as $\omega_{b/i}$, as demonstrated in [5], [6] as follows,

$$
T = m\frac{dV}{dt_i} = m\big(\frac{dV}{dt_b} + \omega_{b/i} \times V\big) \tag{5}
$$

Likewise, the equation representing the rotational motion of a quadrotor under the influence of an applied torque, denoted as τ, can be formulated as follows:

$$
\tau = \frac{dh}{dt_b} + \omega_{b/i} \times h \tag{6}
$$

where h denotes angular momentum.

Hence, the overall dynamics, which are characterized by an upward thrust of Tz and are regarded as having no significant aerodynamic forces, can be described as follows, as stated in [5], [7], [8].

$$
\begin{aligned}
\ddot{x} &= (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{T_z}{m}\\
\ddot{y} &= (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\frac{T_z}{m} - \frac{K_d}{m}\dot{y}^2\\
\ddot{z} &= \cos\phi\cos\theta\frac{T_z}{m} - g\\
\ddot{\phi} &= \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} + \frac{1}{I_x}\tau_\phi\\
\ddot{\theta} &= \dot{\phi}\dot{\psi}\frac{I_z - I_x}{I_y} + \frac{1}{I_y}\tau_\theta\\
\ddot{\psi} &= \dot{\phi}\dot{\theta}\frac{I_x - I_y}{I_z} + \frac{1}{I_z}\tau_\psi
\end{aligned}
\tag{7}
$$

3. Quadrotor Trajectory Tracking Control

Following the development of the quadrotor system dynamics, it is possible to construct the controllers required to sustain the quadrotor along the desired trajectory. Fig. 2 shows the control block diagram for quadrotor UAV trajectory tracking. It is clear that the quadrotor is assisted in tracking the intended trajectory by position and attitude controllers. The position controller generates the thrust control signal, as well as the desired roll and pitch angles. Apart from roll and pitch commands, the attitude controller receives a yaw command from the external input and generates a corresponding control signal to maintain the quadrotor at the intended orientation. The actual trajectory of the quadrotor flight can be obtained using sensors and compared to the desired trajectory, and the controllers generate the control signal to cancel out the error between the two, allowing the quadrotor to follow the intended trajectory.

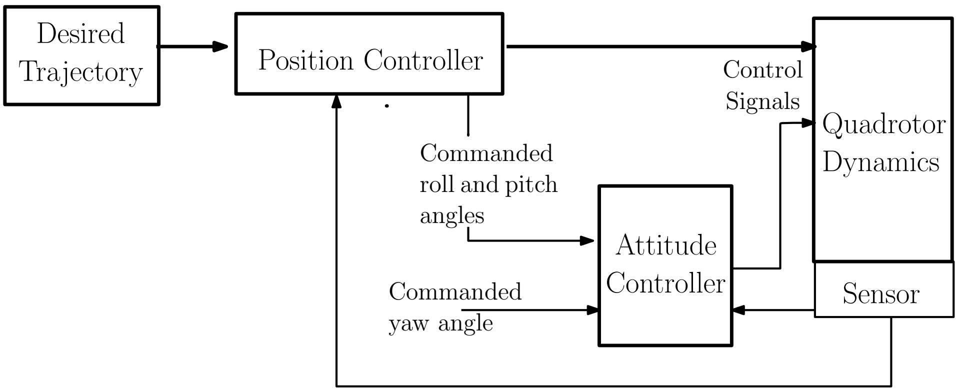

Fig. 2. Block diagram for tracking the desired trajectory of a QUAV

A concise literature review on different controllers for the aforementioned action is provided here with some comprehension.

3.1. Proportional-Integral-Derivative Control

Proportional-integral-derivative (PID) controllers are frequently being deployed in small-scale unmanned aircraft because of its simplicity and ease of tuning [9]. The major job of the controllers in a quadrotor aircraft system is to keep the aircraft steady and follow it along the planned course. Recursively examining the PID control operation [10] reduces the tracking error. The tracking error, its integral or derivative forms, or a combination of these forms is proportional to the control action's correction factor. If e is the tracking error, then the control variable $u(t)$ acquired by a PID controller with proportional gain K, integral gain K_i, and derivative gain K_d is as follows:

$$u(t) = Ke(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \tag{8}$$

PID controller tuning can be done using the trial-and-error method, the Ziegler-Nichols method, or the Cohen-Coon method [11] . PID controllers commonly suffer from impaired quadrotor control during wind up as a result of excessive control signal and overshooting. As a result, [12] suggests that a nonlinear PID be used. Calculating the gain, on the other hand, is computationally expensive. Reference [13] – [17], provide a new way for overcoming the limits of traditional PID when dealing with nonlinear dynamical systems. Here a fuzzy logic PID controller is proposed in which the membership functions are created for obtaining self-tuning capability. However, the accuracy of a control system modelled using fuzzy logic is entirely dependent on human knowledge of the system, and it also requires more time to run [18]. Another method for creating a self-tuned PID controller is to use a neural network to tune the PID controller's gains, which works well in the presence of noise. In addition to standard PD position control, reference [19] added three back-propagated neural network tuned PD controllers for attitude control. Reference [20] uses a neural network-based finite impulse estimator to supply coefficients for a fractional order PID controller in order to improve the performance of the control system. However, there is no exact formula for finding the network's ideal topology, and the time required to reach optimal results is often uncertain [21]. In order to attain the

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optimum tracking performance, evolutionary algorithms are used in the tuning process. In reference [22] and [23], particle swarm optimisation was utilised in conjunction with PID tuning to achieve attitude and altitude control of the quadrotor. To fine-tune the PID controller for quadrotor aircraft, reference [24] and [25] use metaheuristic optimization algorithms including genetic, cuckoo-search, evolutionary, particle swarm optimization, firefly optimization, differential evolutionary algorithms and so on.. The objective of achieving energy efficiency in quadrotor control through the utilization of differential evolution in conjunction with PID control is examined in reference [26]. This study begins by determining PID coefficients using the Ziegler-Nichols method and trial-and-error approach. The PID coefficients are subsequently trained using metaheuristic algorithms.

3.2. Linear Quadratic Regulator

The linear quadratic regulator (LQR) is a modern optimum control method that can provide feedback control while optimizing the objective functions [27] and provide stable behaviour [28]. LQR makes use of the quadrotor aircraft's state space paradigm, assuming that all states are measurable and observable [29], [30]. The linear state space model of the system with state vector $X(t)$, input $U(t)$ and output $Y(t)$ is defined as follows:

$$\begin{aligned}\dot{X} &= AX(t) + BU(t)\\ Y &= CX(t) + DU(t)\end{aligned} \tag{9}$$

where A, B, C, and D represent the system matrix, input matrix, output matrix and feedthrough matrix, respectively. The quadrotor flight through space is defined by a total of twelve states and four control inputs. The states of the quadrotor correlate to the position $(x,\, y,\, z)$, velocity $(v_x,\, v_y,\, v_z)$, Euler angles $(\phi,\, \theta,\, \psi)$ and Euler rates $(p,\, q,\, r)$ in 3D space and are expressed in vector form as $X = [x,\, y,\, z,\, v_x,\, v_y,\, v_z,\, \phi,\, \theta,\, \psi,\, p,\, q,\, r]^T$ with the inputs $U = [u_1,\, u_2,\, u_3,\, u_4]^T$. Thus the linear state space model of the quadrotor aircraft can be written as given in the [31].

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{v_x}\\ \dot{v_y}\\ \dot{v_z}\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&\frac{u_1}{m}&0&0&0&0\\ 0&0&0&0&0&0&\frac{-u_1}{m}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ v_x\\ v_y\\ v_z\\ \phi\\ \theta\\ \psi\\ p\\ q\\ r\end{bmatrix} + \begin{bmatrix}0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ \frac{1}{m}&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&\frac{1}{I_x}&0&0\\ 0&0&\frac{1}{I_y}&0\\ 0&0&0&\frac{1}{I_z}\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix} \tag{10}$$

where m indicates the quadrotor's mass and the moment of inertia is represented by J_x, J_y and J_z. Reference [32] described the lqr feedback control based on this system model, with u_1 equal to mg in the x and y directions. The states of the quadrotor were dissociated into three portions in [33], with one component controlling the attitude, the other the altitude, and the third part directing the position. Attitude control is directly related with the control signals u_2, u_3, and u_4, while altitude control is directly associated with u_1. The position control has an indirect relationship with u_1 through two virtually defined control signals U_x and U_y. These controllers can be characterized using the feedback control law, as mentioned in [34].

$$U = -KX(t) \tag{11}$$

The feed back gain K is calculated using the Hamiltonian function,

$$K = R^{-1}B^TP \tag{12}$$

and P is calculated using the Algebraic Riccati formula as in [35];

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{13}$$

where Q and R are the positive-semi definite and positive definite matrices corresponding to the weights on the states and control inputs in the cost function, respectively. The cost function can be defined as in [36], [37]

$$J = \int_0^\infty (X^T QX + U^T RU)dt \tag{14}$$

The stability range of the LQR controller is significantly limited, as stated in [38], [39]. Consequently, their research employs integral action to propose LQR-I control, and the unscented Kalman filter is utilised for the state estimation method. However, the consequences of model incompatibilities can be further intensified through the incorporation of integral action, potentially leading to instability or a decline in performance. Critical to the performance of LQR are the parameters that dictate its operation; their selection is typically achieved via laborious, iterative trial-and-error processes [40].

3.3. H_∞ Control

In situations characterized by unpredictability or fluctuating operational circumstances, the H_∞ control method is a dependable approach that ensures peak performance and is specifically engineered to attain stability [41], [42]. In order to accomplish this, a controller must be devised that minimizes the "H_∞ norm," which denotes the significant energy gain of the system [43]. The formulation of the control problem is an initial step in the development of a H_∞ control system. The input signals to a system are designated as exogenous input w and manipulated variable u, while the output signals are represented by the measured variable (v) and the required minimizing error signal (z). By noting the feedback gain matrix as K and the system matrix as P, the system can be mathematically represented as

$$\begin{bmatrix} z \\ v \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} \implies z = F_l(P, K)w \tag{15}$$

where $F_l(P, K)$ is the lower linear fractional transformation. The objective of H_∞ control is to find a controller K such that $F_l(P, K)$ is minimized. The infinity norm of $F_l(P, K)$ is formally defined as,

$$||F_l(P, K)||_\infty = \sup_w \overline{\sigma}(F_l(P, K)j\omega) \tag{16}$$

where $\overline{\sigma}$ represents the maximum singular value of $(F_l(P, K)j\omega)$. This problem should encompass a range of criteria, such as efficiency, stability, unpredictability, and disturbance detection [44]. In contrast, uncertainty modelling is an essential component of the control system design process. A radial basis neural network is applied to simulate the unresolved nonlinear characteristics of the standard quadrotor model so as to circumvent the difficulty associated with achieving flawless mathematical modelling [45]. Training neural networks with a radial basis is, nevertheless, a laborious process. For controlling the rotational motion of a quadrotor, a nonlinear H_∞ controller is suggested in [46]. Sliding mode control combined with a nonlinear H_∞ controller is recommended for quadrotor control, according to [47]. But it is probable that the process of fine-tuning the variables of a nonlinear H_∞ controller will be difficult.

3.4. Model Predictive Control

Model predictive control (MPC) is an optimization-based advanced control that can predict the system's future behaviour. It generates control output based on the solution of the optimization problem of interest. As given in [48], the operation of MPC can be divided into three stages.

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- Firstly, the cost function of the optimization problem to solve can be defined as,

$$J(\hat{x}, u) = \int_0^T l(x(t; \hat{x}, u))dt \tag{17}$$

  which is minimized for computing control input.

- Formulate feedback control law.
- Keep updating the current state.

The objective function for quadrotor trajectory tracking control is to reduce tracking error and control effort to the absolute minimum [49]. Consider a quadrotor aircraft having control input $U(k)$ and a state $X(k)$. If the state error $X_e(k)$ at any instant k is defined as the deviation of the current state from the desired state, then MPC uses the incremental objective function as described in reference [50].

$$J_k = \sum_0^{N-1} X_e(k+i|k)^T Q X_e(k+i|k) + U(k+i|k)^T P U(k+i|k) \tag{18}$$

where N is the maximum steps in horizon, Q and R are the positive definite weight matrices. In reference [51], the operating concept of MPC is described. MPC forecasts the system's future behaviour for a finite prediction horizon Np using the measurements taken at instant k and the system model. The control input is then decided as the preset optimization task is completed across a range Nc less than Np. The cost function in reference [52] takes control action into account as well as;

$$J_k = \sum_0^{N_p-1} X_e(k+i|k)^T Q X_e(k+i|k) + \sum_0^{N_c-1} \Delta U(k+i|k)^T R \Delta U(k+i|k) + \sum_0^{N_p-1} U_e(k+i|k)^T N U_e(k+i|k) \tag{19}$$

Here, the prediction horizon is set at between 0 and $N_p - 1$, and the control horizon lies in the interval $(0, N_c - 1)$. The weight matrices corresponding to state, rate of change of control and control action are given by Q, R and N, respectively.

Reference [53] improves the controller's performance by combining steady state optimal MPC control law and Laguerre optimal MPC control law. A nonlinear MPC controller is developed in [54] to address the control action of a quadrotor considering its nonlinear dynamics. The nonlinear MPC in [55] and [56] assume that the input applied to the system is in the form of;

$$u(t) = k\hat{x}(t) \tag{20}$$

the closed loop gain k is found using discrete Riccati equation;

$$k = (APC^T + Z)(CPC^T + V)^{-1} \tag{21}$$

where A and C are the state matrix and output matrix respectively. V is the Gaussian distributed zero mean measurement noise and Z is the cross covariance of state and measurement noise.

3.5. Feedback Linearization Control

Feedback linearization control is a powerful approach that allows the application of feedback control methods by algebraically transforming the dynamics of a nonlinear system into a completely or partially linearized system [57]. Reference [58] uses a fully linearized representation of a nonlinear system and a shifting feedback linearization controller to prevent saturation and preserve stability in confined linearized systems. A nonlinear system's feedback linearization control scheme is outlined in [59] and [60] as follows;

$$\begin{aligned} \dot{x} &= f(x) + \sum_{j=1}^{m} g_j(x) u^{(j)} \\ y &= [y_1, y_2, ..., y_m] \end{aligned} \tag{22}$$

x, $u^{(j)}$, f, f_j are vector -valued functions can be decoupled to a linear system.

$$\begin{aligned} y_1^{(r_1)} &= v^{(1)} \\ y_2^{(r_2)} &= v^{(2)} \\ &\vdots \\ y_m^{(r_m)} &= v^{(m)} \end{aligned} \tag{23}$$

using state feedback control,

$$u = \Delta^{-1}(-b + v) \tag{24}$$

where Δ is the decoupling matrix given by

$$\Delta = \begin{bmatrix} L_{g_1}^1 L_f^{(r_1-1)} y_1 & L_{g_2}^1 L_f^{(r_1-1)} y_1 & \dots & L_{g_m}^1 L_f^{(r_1-1)} y_1 \\ L_{g_1}^1 L_f^{(r_2-1)} y_2 & L_{g_2}^1 L_f^{(r_2-1)} y_2 & \dots & L_{g_m}^1 L_f^{(r_2-1)} y_2 \\ \vdots & \vdots & \vdots & \vdots \\ L_{g_1}^1 L_f^{(r_m-1)} y_m & L_{g_2}^1 L_f^{(r_m-1)} y_m & \dots & L_{g_m}^1 L_f^{(r_m-1)} y_m \end{bmatrix} \tag{25}$$

and b is given by

$$\begin{bmatrix} L_f^{r_1} y_1, & L_f^{r_2} y_2, & \dots, & L_f^{r_m} y_m \end{bmatrix}^T \tag{26}$$

where $L_f^k y_j$ is the k^{th} Lie derivative of y_j. The unmanned quadrotor aircraft has six degrees of freedom and four inputs of control. The control input vector can be written as $u^{(1)}$, $u^{(2)}$, $u^{(3)}$, and $u^{(4)}$, where $u^{(1)}$ is the underactuated control signal that can be delayed by a double integrator [61]. Reference [62] achieves quadrotor attitude control using feedback linearization control, and the adaptive attitude control is proposed in [63] using a combination of model reference adaptive control and feedback linearization to ensure that it can operate effectively in the presence of parameter variations and gust disturbances. However, this methodology could potentially necessitate further intricacy for time-varying systems and might not consistently ensure satisfactory performance. An article [64] proposed combining PID controllers with feedback linearization and feed-forward control using backstepping to control both the attitude and position of quadrotors.. For this purpose, a simpler nonlinear system model is evaluated; however, performance may degrade in the event of parameter uncertainties. Integration of neural networks and disturbance observer-based feedback linearization is proposed in [65] as a solution to the issue of quadrotor destabilization caused by disturbances. In neural network based feedback linearization control, the weights of the neural network are modified in real-time [66]. This process may take a considerable amount of time.

3.6. Backstepping Control

Backstepping is a recursive method of developing nonlinear controllers for dynamical systems that rely on the Lyapunov theorem to ensure system stability [67], [68]. Let's look at the backstepping controller design using the quadrotor system model from [69] as an example. Here, the state vector X is thought to be made up of six variables that correlate to the location and orientation ($X = [x_1, x_2, x_3, x_4, x_4, x_5, x_6]^T = [x, y, z, \phi, \theta, \psi]^T$), and the control inputs $u = [u_1, u_2, u_3, u_4]^T$. Thus, the state space model can be written as follows;

$$\begin{aligned} \ddot{X} &= f(X) + g(X)u \\ \ddot{X} &= \begin{bmatrix} 0 \\ 0 \\ -g \\ \dot{\theta}\dot{\psi}a_1 - \dot{\theta}a_2\Omega_d \\ \dot{\phi}\dot{\psi}a_3 + \dot{\phi}a_4\Omega_d \\ \dot{\theta}\dot{\phi}a_5 \end{bmatrix} + \begin{bmatrix} \frac{u_x}{m} & 0 & 0 & 0 \\ \frac{u_y}{m} & 0 & 0 & 0 \\ \frac{u_z}{m} & 0 & 0 & 0 \\ 0 & b_1 & 0 & 0 \\ 0 & 0 & b_2 & 0 \\ 0 & 0 & 0 & b_3 \end{bmatrix} u \end{aligned} \tag{27}$$

where $a_1 = \frac{(I_y - I_z)}{I_x}$, $a_2 = \frac{J_r}{I_x}$, $a_3 = \frac{(I_z - I_x)}{I_y}$, $a_2 = \frac{J_r}{I_y}$, $a_5 = \frac{(I_x - I_y)}{I_z}$, $a_4 = \frac{J_r}{I_z}$, $b_1 = \frac{l}{J_x}$, $b_2 = \frac{l}{J_y}$, $b_3 = \frac{1}{J_z}$, J_r is the rotor inertia, $u_x = \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi)$, $u_x = \cos(\phi)\sin(\theta)\sin(\psi) -$

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$\sin(\phi)\cos(\psi)$, $u_z = \cos(\phi)\cos(\theta)$, Ω_i, $i = 1, 2, 3, 4$ be the angular velocities of rotors and $\Omega_d = \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3$.Due to the fact that backstepping is a recursive process, the controller design goes through numerous stages. Let's use this technique from [70] and [71] to regulate the altitude of a quadrotor.

Step1: The altitude tracking error is denoted as $e_1 = z_d - z$, (desired altitude minus actual altitude) and is stabilised using Lyapunov function $V_1(e_1) = \frac{1}{2}e_1^2$. The derivative of the chosen Lyapunov function should be negative definite in order to guarantee stability. Thus,

$$\dot{V}_1(e_1) = e_1\dot{e_1} = e_1(\dot{z_d} - \dot{z}) \tag{28}$$

The next stage of design involves choosing a stabilizing function for the e_1. Let it be $\alpha_1 = \dot{z_d} + K_1 e_1$. In this step α_1 is selecting to be equal to $\dot{(z)}$. Thus, $\dot{V}_1(e_1) = -K_1 e_1^2 \leq 0$ for positive values of K_1.

Step 2: The second tracking error e_2 can be defined as the difference between $\dot{z}$ and α_1. Thus

$$\begin{aligned} \dot{e_2} &= \ddot{z} - \dot{\alpha_1} \\ \dot{e_2} &= f(x_3) + g(x_{31})u_1 - \ddot{z}_d - K_1\dot{e_1} \end{aligned} \tag{29}$$

A second Lyapunov function can be used as $V_2 = \frac{1}{2}(e_1^2 + e_2^2)$ to stabilise e_2, and $\dot{V}_2$ must be less than or equal to zero.

$$\dot{V}_2 = e_1\dot{e}_1 + e_2\dot{e}_2 \tag{30}$$

Substituting for $\dot{e}_1$ and $\dot{e}_2$ and define control law u_1 in (30) as,

$$u_1 = \frac{1}{g(x_{31})}(e_1 - f(x_3) + \ddot{z}_d + K_1\dot{e_1} - K_2 e_2) \tag{31}$$

makes $\dot{V}_2 \leq 0$ Thus, the quadrotor's control law u_1 is established. Finding the remaining control laws, u_2, u_3, and u_4, requires similar processes.

The adaptive backstepping control method is illustrated in [72] – [77] and it enables the quadrotor to function efficiently in the presence of external disturbances and uncertainties like mass and inertial variations. According to reference [78], a quadrotor backstepping control is suggested using online optimization as a foundation. Here, a problem of optimization is devised to deal with both local motion planning and cascaded tracking control action. The construction of backstepping control using radial basis function neural network is described in [79] – [82]. Thus, despite uncertainty, the quadrotor aircraft can follow the desired trajectory. Due to the real-time adjustment of the neural network's weights, it is conceivable that an extended duration may be necessary in this particular situation.

3.7. Sliding Mode Control

Sliding mode control (SMC) is one of the most efficient and robust nonlinear control techniques for sophisticated nonlinear dynamical systems [83], [84]. During the controller's design phase, it decouples higher-dimensional systems into smaller subsystems [85]. For the sliding mode controller design, the disturbance effect is taken into account in the state space model as described in [86],

$$\dot{X} = f(X, U, d) \tag{32}$$

where X represents state, U is the input and d is the disturbance. SMC is a two-step procedure that begins with the design of the sliding surface and continues with the construction of the controller in such a way that the system dynamics is pushed to slide over the sliding surface [87]. There are numerous ways to describe the sliding surface. The sliding surface and control law design for a quadrotor system using a conventional sliding mode control is presented in [88],

$$\begin{aligned} S_i = \dot{e}_i + \alpha_i e_i, \quad \alpha_i > 0 \\ U = -K sign(S_i) \end{aligned} \tag{33}$$

where $i = x, y, z, \phi, \theta, \psi$ describes the six degrees of freedom of the quadrotor and e is the error in the desired state to the actual one. Here K is the controller gain. In an effort to enhance the performance and robustness of conventional SMC, [89] – [92] presents the fractional order sliding mode as a potential solution. Chattering is a common problem with traditional sliding mode control, which can be alleviated by employing integral-backstepping combined sliding mode control [93]. In [94], terminal integral based sliding mode control is employed to achieve rapid and precise quadrotor trajectory tracking. Here the sliding surface is defined as follows,

$$S_i = e_i + \int \alpha_i e_i^{\frac{q_i}{p_i}} + \beta_i e_i^{(q_i(2p_i - q_i))} dt \tag{34}$$

where p and q are positive integers and α and β are positive constants and exponential reaching laws can be used as

$$\dot{S}_i = -\lambda_i S_i - K_i Sgn(S_i) \tag{35}$$

where λ and K are the tuning parameters. These tuning parameters are determined by an adaptive approach, which results in adaptive sliding mode control [95] and [96]. The papers [96] – [101] provide a detailed explanation of how an adaptive sliding mode controller is implemented. It also introduces an adaptive control rule that has been designed to promote rapid convergence and minimize the steady-state error to a more practical level. When constructing a sliding surface, adaptive laws are utilized in [102] – [105] to generate an adaptive sliding mode controller. The utilization of second-order sliding mode control for quadrotor control is demonstrated in reference [106], while reference [107] outlines the application of higher-order sliding mode control for quadrotor control. The system model has a significant impact on sliding mode control. When used for a nonlinear underactuated quadrotor system, unmodeled dynamics play a significant role in real-world situations. References [108] and [109] suggest a solution to this problem: a model-free, single-dimension fuzzy-based sliding mode control that can track the desired trajectory even in the face of unmodeled dynamic forces. For the purpose of adapting to uncertainty and unknown disturbances, the adaptive sliding mode controller is described in [110] – [115], which is implemented by means of a neural network. However, it is worth noting that the computation of fuzzy logic operations and the ongoing switching of the sliding mode control can both spike up processing power requirements.

3.8. Fuzzy Controller

The accuracy of the controller design addressed in earlier parts depends on how thoroughly the model is defined. Since building models for aerial vehicles is a laborious and time-consuming operation [116], model-free control designs offer a simpler option. Fuzzy control is a knowledge-based, model-free controller that can manage nonlinear systems [117]. The fuzzification, rule base, inference engine, and defuzzification sub-modules are the four main components of the fuzzy inference system [118].

The fuzzification unit takes the crisp input data set and turns it into a fuzzy set [119]. Fuzzy rules are kept in a fuzzy rule base (IF-THEN rules). Furthermore, it contains membership functions for providing the input variables for the fuzzy rule base and the system output variables. A fuzzy logic controller's rule base contains two inputs: error e(t) and change in error $\Delta(e(t))$. The error is produced by the difference between the quadrotor's actual state and desired state, for instance, the altitude error is $e_z(t) = z - z_d$ and the change in error is $\Delta(e_z(t)) = \dot{z} - \dot{z_d}$ as in [120]. Rules must be defined in such a way to produce more exact results because they heavily rely on human understanding while being defined. Let's have a look at the rule base described in [121] with five membership functions, negative large (NL), negative small (NS), zero (Z), positive small(PS) and positive large (PL) and five membership sets for the controller output as negative large (NL), negative small (NS), zero (Z), Medium (M) and Big (B), then the rules are described as follows;

IF $e(t)$ is NL and $\Delta(e(t))$ is NL, **THEN** the output is Z
IF $e(t)$ is Z and $\Delta(e(t))$ is Z, **THEN** the output is M

Create each rule according to the guidelines in [122], which are listed in the Table 1

The decision-making centre of fuzzy control is the inference engine, which uses fuzzy rules stored in the rule-base to make judgments. Fuzzy system operation is supported by the selection of fuzzy rules [123]. The defuzzifier transforms the fuzzy data set into output data that is crisp, which is the control input to the quadrotor system. Mamdani FIS and Sugeno FIS are the two fuzzy inference systems (FIS) available. It requires a Mamdani FIS if the rule base is set up as described above with a fuzzy output configured. Reference [124] enhances the proportional-derivative autopilot's performance using Mamdani FIS. It correlates to Sugeno FIS if the fuzzy rule's output is a crisp value. Sugeno FIS is used for quadrotor control in [125] and [126].To improve transient performance, references [127] – [132] combine a hybrid controller made from PID with an adaptive neuro-fuzzy inference system. Conversely, the weak theoretical underpinnings of specific domains of fuzzy logic pose challenges in the formalisation and verification of stability and convergence properties.

Table 1. Fuzzy rule creation

		e(t)				
		NL	**NS**	**Z**	**PS**	**PL**
$\Delta(e(t))$	NL	Z	Z	Z	S	M
	NS	Z	Z	S	M	M
	Z	Z	S	M	M	M
	PS	S	S	M	B	B
	PL	M	M	M	B	B

3.9. Model Reference Adaptive Controller

The model reference adaptive controller (MRAC) has the flexibility to cope to the real-time performance of quadrotor unmanned aircraft despite parameter uncertainty [133], [134]. This adaptation is achieved by integrating a reference model and an adaptive mechanism while designing a feedback control law [135]. Create an initial reference model for the system's optimal response characteristics, such as,

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{36}$$

with reference state vector x_m, reference state matrix A_m, reference input matrix B_m and reference input signal $r(t)$. After that, combine this reference model, the adaptive mechanism, and the real system to create a closed loop control system. The tracking error is the result of the comparator in closed loop control, which is interpreted as the difference between the outputs of states of the actual system (x) and reference model (x_m),

$$e(t) = x(t) - x_m(t) \tag{37}$$

This error is minimised as much as possible by incorporating an adjustive mechanism into the control loop. Various adjusting techniques are used, such as the Lyapunov-based adjustment mechanism, in which a Lyapunov function is defined as a function of error and the update laws are set in such a way that the time derivative of the Lyapunov function is negative definite [136] – [138]. Another approach is to use the MIT rules based on the gradient method. To improve tracking performance, Reference [139] employs an MRAC structure and adjustment mechanisms that employ a combination of MIT rules and conventional, integral, and higher order sliding mode controls. References [140] and [141] proposed a deep neural network-based MRAC in which the network is learned to minimise tracking error. A substantial discrepancy between the reference model and the real-world plant environment may potentially result in a decline in the performance of the control system.

3.10. L1 Adaptive Controller

L_1 adaptive controller is a reliable adaptive control method that partially compensates for uncertainty. Knowing that L_1 norm is the induced L_∞ norm of the input/output signal, the bounded performance of the input/output signals can be satisfied by the adequate condition for stability expressed in terms of L_1 norm. With the addition of a low pass filter in the control loop, L_1 adaptive control substitutes the MRAC with its structural equivalent. The system may adapt quickly and become more robust with the addition of low pass filter (LPF) [142]. The essential components of L_1 control are the state predictor, adaptation law, control law with low pass filter, and a reference model [143] – [146]. The functionality of the L_1 controller can be explained by considering the state model in [147], where $x(t)$, $u(t)$, and $y(t)$ represent the state vector, control input, and output vector, respectively, and are influenced by time varying uncertainties $\gamma(t)$ and matched disturbances $\sigma(t)$ as,

$$\begin{aligned} \dot{x}(t) =& x(t) + B\Big(\gamma(t)x(t) + wu(t) + \sigma(t)\Big) \\ y(t) =& C^T x(t); \qquad x(0) = x_0 \end{aligned} \tag{38}$$

where A is the Hurwitz matrix, w is the constant nonsingular unknown matrix, B and C are constant matrix. The states of the reference model output and the actual system output are anticipated by the state predictor using the w matrix as the identity matrix as follows,

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\Big(\hat{\gamma}(t)x(t) + u(t) + \hat{\sigma}(t)\Big) \tag{39}$$

and these predicted states are then sent to the adaption law block. The adaption law is intended to produce more exact tracking results. Reference [143] establishes adaptation law based on projection operation and achieves geometric attitude control of the quadrotor as $\text{proj}_\Gamma\big(\text{predicted uncertainty } \hat{\gamma}(t), f\big)$ with f is a function of state error, $\tilde{x}(t) = \hat{x}(t) - x(t)$. The output of the adaptation block is coupled to a feedback law module that has a low pass filter, providing the quadrotor system with the necessary correction factor to follow the desired trajectory. For the purpose of accommodating the unavoidable disturbances that the quadrotor endures while travelling a predefined path, L1-optimal control is implemented in [148]. Although L1 optimal control exhibits resilience to disturbances and uncertainties, it may not offer explicit assurances regarding performance or stability in all circumstances. In reference [149], the backstepping creation of a quadrotor trajectory tracking control is integrated using the L1 control approach. This alteration empowers the system to attain closed-loop functionality that is not only applicable on a global scale but also unique. However, the implementation of backstepping in L1 adaptive control might necessitate intricate mathematical computations and analyses.

3.11. NARMA L2 Controller

Nonlinear auto regressive moving average (NARMA) is a straightforward and efficient discrete-time model of a nonlinear system. The most prevalent NARMA models are the NARMA L1 and NARMA L2. Since the NARMA L2 model is easier to implement, reference [150] states that it is recommended for designing tracking controllers. The NARMA L2 model for a nonlinear system with control input u[k] is described in reference [151] as follows:

$$\begin{aligned} y[k+d] =& f_0[y(k), y(k-1), \ldots + y(k-n+1), u(k), u(k-1) \\ & + \ldots + u(k-n+1)] + g_0[y(k), y(k-1), \ldots, \\ & y(k-n+1), u(k-1), \ldots + u(k-n+1)]u(k) \end{aligned} \tag{40}$$

The NARMA L2 neural controller design involves two phases- system identification and control design [152]. The initial phase is the approximation of functions f and g using two neural subnetworks [153]. The structure of NARMA L2 controller is described in [154] is the realization of control law $u(k)$ for a desired output $y_r[k+d]$ and the control law $u(k)$ can be derived from (40) as,

$$\begin{aligned} N(k) =& \Big(y_r[k+d] - f[y(k), y(k-1), \ldots + y(k-n+1), \\ & u(k), u(k-1) + \ldots + u(k-m+1)]\Big) \\ D(k) =& \big(g[y(k), y(k-1), \ldots, y(k-n+1), \\ & u(k-1), \ldots + u(k-m+1)]\big) \\ u(k) =& \frac{N(k)}{D(k)} \end{aligned} \tag{41}$$

The adaptive NARMA L2 controller is described in [155] using online state vector regression. It can be challenging to construct a nonlinear auto-regressive moving average model that accurately represents the intricate dynamics of the system. The functionality of the NARMA L2 controller is highly dependent on the precision of the model. When uncertainties or disturbances are present, inconsistencies or errors in the model may result in suboptimal control performance or instability.

4. Discussion and Comparative Analysis

Even though the controller's role in the autopilot is to guide the quadrotor aircraft along the desired course, each controller performs differently because they each have unique qualities. The most appropriate controller must be selected with correct design in accordance with the application context. The assessment evaluates the strengths and weaknesses of different control systems in terms of their ability to handle uncertainty, maintain system stability, and withstand disturbances. In cases where the system is linear and well-understood, a PID or LQR controller may suffice, but non-linearities and uncertainties may necessitate the use of controllers such as SMC or MPC, or adaptive strategies such as L1 adaptive control and MRAC. Another crucial component is computational resources. As an illustration, MPC requires a significant amount of computational power due to its online optimization, whereas simpler controllers like PID or LQR require less computing power. A further significant factor to take into account is robustness to uncertainties and disturbances. Better flight performance in a disturbed environment is generally provided by SMC and adaptive control techniques. In addition, the presence of state constraints can potentially assist with MPC or feedback linearization. It's important to keep in mind that while PID controllers are simple to implement, more complex techniques like NARMA or MPC may require significant tuning operations. The rigorous mathematical framework provided by H-infinity control makes it possible to create reliable controllers that are capable of managing system uncertainties and disturbances. Choosing H-infinity control requires careful consideration of a number of factors, including the availability of precise models of the UAV dynamics, the degree of environmental uncertainty, and the desired degree of robustness against disturbances. Fuzzy controllers, on the other hand, provide a more flexible and intuitive approach to control, particularly in situations where the system's dynamics are highly nonlinear or difficult to accurately model. When considering fuzzy controllers, researchers should consider their level of system dynamics knowledge, the ease with which control decisions can be interpreted, and the uncertainty of the environment. In order to select the best controller among the options available, researchers must ultimately compare these factors to the unique needs and constraints of their quadrotor application.

The advantages, disadvantages, and modifications of each controller are listed in Table 2. The performance limitations of linear classical controllers are restricted to the area surrounding the equilibrium point under evaluation. The stable configurations of the quadrotor UAV are possible due to its nonlinear nature. In order to attain maximum efficiency, it is imperative to construct multiple controllers through the process of linearizing the system's dynamics at each equilibrium point. Moreover, performance degrades with the distance that state values deviate from their equilibrium points. The preceding observations underscore the significance of employing nonlinear controllers in order to attain maximum efficiency and efficacy in the operation of the controller.

Table 2. Comparison of different controllers

Controller	Merits	Demerits	Hybrid versions
PID	robust to wide range of payload, easily implementable	overshoot, slow oscillations, noise amplification	Nonlinear PID, fuzzy PID, NN based PID
LQR	provide stable behaviour, optimal for linear systems	vulnerable to inaccuracies in the modelling, intensive computational tasks	LQG, LQR-I control, LQR with Kalman filter
H_∞ controller	robustness in the face of uncertainty, rejection of disturbances	extensive computational demands, intricate design	adaptive H_∞ control, nonlinear H_∞ control
MPC controller	effectively manages constraints, capability for predicting	heavy computational demand, real-time control is extremely complex	MPC with PID, adaptive MPC
Feedback Linearization	precise tracking, controls nonlinear systems	vulnerable to uncertainties, precise system modelling is essential	fuzzy-FLC, adaptive FLC
Backstepping controller	robust, effective for highly nonlinear systems	increased system complexity, results in greater complexity	adaptive backstepping
Sliding mode Controller	robust to disturbances, handles uncertainties to an extend	chattering effect, intense control effort	adaptive SMC, fuzzy-SMC
Fuzzy controller	capable to handle uncertainties, linguistic rules	limited theoretical concepts, tuning complexity	neuro-fuzzy controller
Model reference adaptive control	adaptability, robust	accuracy depends on how accurate reference model is	MRAC with PID, MRAC with LQR
L_1 adaptive controller	robust, fast adaptation	computationally intensive complex	L_1+PID, L_1+MPC
NARMA L_2 controller	Good for nonlinear systems	Limited to specific applications modelling complexity	NARMA L_2 fuzzy control

5. Future Directions

The next generation of research on quadrotor UAV control offers both promising avenues for exploration and difficult problems that will require multidisciplinary approaches to solve present problems and uncover new avenues for investigation. A viable approach involves incorporating adaptive elements into conventional PID controllers, utilising the versatility of adaptive methods and the resilience of PID control to efficiently manage ambiguities and dynamic surroundings. A comprehensive strategy that combines the accuracy of mathematical models with the adaptability and interpretability of fuzzy logic can also be provided by combining model-based controllers with fuzzy logic systems. One key strategy for enhancing the adaptability and endurance of control strategies is to combine principles of control with robotics as well as artificial intelligence (AI). In order to facilitate autonomous development of complex control policies and enable UAVs to adapt to dynamic environments and unplanned scenarios, researchers can look into reinforcement learning algorithms. Furthermore, integrating control theory with AI methods like computer vision and deep learning can aid in real-time perception and making decisions. This will improve the accuracy and autonomy of UAV navigation in congested and turbulent environments. Furthermore, investigating bio-inspired control strategies, which are based on the collective behaviour of natural systems, may provide new insights into swarm adaptability and synchronisation, as well as pave the way for the development of autonomous control algorithms for quadrotor networks. The development of more resilient, flexible, and intelligent quadrotor UAV control systems will require interdisciplinary approaches from control theory, artificial intelligence, robotics, and other pertinent fields. Doing so could drastically alter the potential uses and implications of quadrotor UAV technology in the future.

6. Conclusion

This paper provides a comprehensive analysis of control algorithms for quadrotor trajectory tracking, shedding light on the diverse approaches that can be employed to ensure precise and manoeuvrable control of these unmanned aerial vehicles. This discussion encompasses a spectrum of control systems, commencing with traditional methodologies like PID and LQR controllers and progressing to contemporary strategies including MPC, H-infinity, and adaptive control. Every algorithm possesses unique strengths with regard to simplicity, adaptation, or flexibility in response to varying operational circumstances. The ability to withstand disturbances, uncertainties, and adapt to changing conditions is crucial for controlling quadrotors. Algorithms like adaptive control, sliding mode control, and certain hybrid approaches exhibit considerable resilience to uncertainties, albeit at the expense of added intricacies.

The analysis reveals that the combination of various control techniques, such as integrating adaptive components with PID or merging model-based controllers with fuzzy logic, has demonstrated promise in improving overall performance and robustness while addressing limitations of individual algorithms. Quadrotors still face challenges in achieving precise trajectory tracking control, even with notable progress. These challenges primarily involve managing complex dynamics, meeting real-time computational demands, and ensuring robustness in uncertain environments. Understanding the real-world constraints that researchers encounter when applying sophisticated control algorithms is essential for successfully navigating the challenging field of quadrotor UAV control. These challenges cover practical issues like sensor noise, actuator limitations, and real-world implementation challenges, and go beyond handling complex dynamics and guaranteeing robustness in unpredictable environments. Furthermore, quadrotor UAVs' agility and responsiveness may be limited by actuator constraints, such as motor saturation and response time, especially in dynamic flight scenarios.

Furthermore, problems like hardware constraints and energy consumption could make some control strategies impractical for real-world UAV applications. Addressing these practical limitations requires careful consideration of system constraints, robust control design, and optimization of computational resources to ensure reliable and efficient operation in real-world environments. Potential areas for future research could centre around the advancement of hybrid or learning-based methodologies, utilizing artificial intelligence and machine learning techniques to enhance adaptability and optimize performance. Through the use of AI algorithms, the UAVs are able to learn intricate behaviours, adjust to changing environments, and instantly optimize control schemes. Furthermore, new avenues for improving UAV manoeuvrability, navigation, and environment interaction can be opened up by incorporating robotics principles to control systems. Adopting multidisciplinary methods can help to create more resilient, flexible, and intelligent control schemes that have the potential to completely transform the uses and capabilities of quadrotor unmanned aerial vehicles (UAVs) across a range of industries.

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